MEC6503 Industrial Robotics
Project 1- Postures for a Fanuc M16iB manipulator

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ABSTRACT

A Fanuc M16iB [1] manipulator is used to deburr the edge point of a spherical block with a tool. In a first step, the direct kinematics of the robot are computed. Then the inverse kinematics problem is solved to calculate the joint angles needed to let the tool tip point perpendicular at the surface of the spherical block at one edge point, which gave four theoretical solutions. Testing on a robot simulator [2] however showed that only one of the solutions is physically feasible. The work thus shows that extreme caution has to be used, when theoretical values are to be implemented in a real robot, as these might lead to hardware damage.

1 INTRODUCTION

The Fanuc M-16ib [1] is a commercially available serial robot used for material handling. The project is to simulate the use of this robot for deburring the edge of a spherical block. In this paper we describe how to pose the manipulator at one specific position of the spherical block. To do so direct and inverse kinematics have to be developed.
The direct kinematics are used to calculate the pose of an end-effector for a given manipulator and a given posture. This is done based on the Davin-Hartenberg (D-H) parameters given for the robot. With the inverse kinematics the posture of a manipulator is calculated given the posture of its end-effector. Verification of the results are done with a simulator [2].

2 DIRECT KINEMATICS

2.1 CALCULATION

The David-Hartenberg parameters of the Fanuc M-16iB [1] described in table 1.

$joint$	θ_i	a_i	b_i	α_i	Min.	Max.
1	θ_1	0.150	0.525	$-\pi/2$	-2.9671	2.9671
2	θ_2	0.770	0.0	0.0	-2.1817	2.1817
3	θ_3	0.10	0.0	$\pi/2$	-4.0143	4.0143
4	θ_4	0.0	0.740	$-\pi/2$	-3.4907	3.4907
5	θ_5	0.0	0.0	$\pi/2$	-2.4435	2.4435
6	θ_6	0	0.10	0	-7.8540	7.8540
$unit$	$rad.$	m	m	$rad.$	$rad.$	$rad.$

Table 1 D-H Parameters of Fanuc M16iB [1],[3]

Using the D-H Parameters, the coordinate transformation matrix of robot are calculated the the following:

$$T = T_1\, T_2\, T_3\, T_4\, T_5\, T_6\, A_{tool} \tag{1}$$

$$\text{Where } T_i = \begin{bmatrix} cos(\theta[i]) & -cos(\alpha[i]) \times sin(\theta[i]) & \sin(\alpha[i]) \times sin(\theta[i]) & a[i] \times cos(\theta[i]) \\ sin(\theta[i]) & cos(\alpha[i]) \times cos(\theta[i]) & -\sin(\alpha[i]) \times cos(\theta[i]) & a[i] \times sin(\theta[i]) \\ 0 & sin(\alpha[i]) & cos(\alpha[i]) & b[i] \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[4]

$$\text{and } A_{tool} = \begin{bmatrix} cos(\beta) & 0 & sin(\beta) & -0.0785 \\ 0 & 1 & 0 & 0 \\ -sin(\beta) & 0 & cos(\beta) & 0.154 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ with } \beta = 25.2\ ^{o} \quad [3]$$

thus giving T=

$$\Big[\ [$$

$$\begin{aligned}
& 0.9048270524 \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \\
& - 0.9048270524 \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \\
& - 0.9048270524 \cos(\theta_6) \cos(\theta_5) \sin(\theta_1) \sin(\theta_4) \\
& - 0.9048270524 \cos(\theta_6) \sin(\theta_5) \cos(\theta_1) \cos(\theta_2) \sin(\theta_3) \\
& - 0.9048270524 \cos(\theta_6) \sin(\theta_5) \cos(\theta_1) \sin(\theta_2) \cos(\theta_3) \\
& - 0.9048270524 \sin(\theta_6) \sin(\theta_4) \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \\
& + 0.9048270524 \sin(\theta_6) \sin(\theta_4) \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \\
& - 0.9048270524 \sin(\theta_6) \sin(\theta_1) \cos(\theta_4) \\
& - 0.4257792917 \sin(\theta_5) \cos(\theta_4) \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \\
& + 0.4257792917 \sin(\theta_5) \cos(\theta_4) \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) \\
& + 0.4257792917 \sin(\theta_5) \sin(\theta_1) \sin(\theta_4) - 0.4257792917 \cos(\theta_5) \cos(\theta_1) \cos(\theta_2) \sin(\theta_3) \\
& - 0.4257792917 \cos(\theta_5) \cos(\theta_1) \sin(\theta_2) \cos(\theta_3), \\
& -\sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \\
& + \sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \cos(\theta_1) \sin(\theta_2) \sin(\theta_3) + \sin(\theta_6) \cos(\theta_5) \sin(\theta_1) \sin(\theta_4)
\end{aligned}$$

$$+\sin(\theta_6)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+\sin(\theta_6)\sin(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)-\cos(\theta_6)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+\cos(\theta_6)\sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)-\cos(\theta_6)\sin(\theta_1)\cos(\theta_4),$$
$$0.4257792917\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-0.4257792917\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-0.4257792917\cos(\theta_6)\cos(\theta_5)\sin(\theta_1)\sin(\theta_4)$$
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$$+0.4257792917\sin(\theta_6)\sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-0.4257792917\sin(\theta_6)\sin(\theta_1)\cos(\theta_4)$$
$$+0.9048270524\sin(\theta_5)\cos(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-0.9048270524\sin(\theta_5)\cos(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-0.9048270524\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+0.9048270524\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+0.9048270524\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3),$$
$$-0.07850000000\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$+0.07850000000\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
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$$+0.07850000000\sin(\theta_6)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-0.07850000000\sin(\theta_6)\sin(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$+0.07850000000\sin(\theta_6)\sin(\theta_1)\cos(\theta_4)$$
$$+0.2540000000\sin(\theta_5)\cos(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-0.2540000000\sin(\theta_5)\cos(\theta_4)\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$-0.2540000000\sin(\theta_5)\sin(\theta_1)\sin(\theta_4)+0.2540000000\cos(\theta_5)\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+0.2540000000\cos(\theta_5)\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)+0.7400000000\cos(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
$$+0.7400000000\cos(\theta_1)\sin(\theta_2)\cos(\theta_3)$$
$$+0.1000000000\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)-0.1000000000\cos(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$+0.7700000000\cos(\theta_1)\cos(\theta_2)+0.1500000000\cos(\theta_1)], [$$
$$).9048270524\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$
$$-0.9048270524\cos(\theta_6)\cos(\theta_5)\cos(\theta_4)\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)$$
$$+0.9048270524\cos(\theta_6)\cos(\theta_5)\cos(\theta_1)\sin(\theta_4)$$
$$-0.9048270524\cos(\theta_6)\sin(\theta_5)\sin(\theta_1)\cos(\theta_2)\sin(\theta_3)$$
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$$+0.9048270524\sin(\theta_6)\cos(\theta_1)\cos(\theta_4)$$
$$-0.4257792917\sin(\theta_5)\cos(\theta_4)\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$+ 0.4257792917 \sin(\theta_5) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3)$$
$$- 0.4257792917 \sin(\theta_5) \cos(\theta_1) \sin(\theta_4) - 0.4257792917 \cos(\theta_5) \sin(\theta_1) \cos(\theta_2) \sin(\theta_3)$$
$$- 0.4257792917 \cos(\theta_5) \sin(\theta_1) \sin(\theta_2) \cos(\theta_3),$$
$$-\sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(\theta_1) \cos(\theta_2) \cos(\theta_3)$$
$$+ \sin(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) - \sin(\theta_6) \cos(\theta_5) \cos(\theta_1) \sin(\theta_4)$$
$$+ \sin(\theta_6) \sin(\theta_5) \sin(\theta_1) \cos(\theta_2) \sin(\theta_3)$$
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$$+ \cos(\theta_6) \sin(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + \cos(\theta_6) \cos(\theta_1) \cos(\theta_4),$$
$$0.4257792917 \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(\theta_1) \cos(\theta_2) \cos(\theta_3)$$
$$- 0.4257792917 \cos(\theta_6) \cos(\theta_5) \cos(\theta_4) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3)$$
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$$+ 0.7700000000 \sin(\theta_1) \cos(\theta_2) + 0.1500000000 \sin(\theta_1)], [$$
$$-0.9048270524 \cos(\theta_6) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \cos(\theta_3)$$
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$+ 0.4257792917 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3) - 0.4257792917 \cos(\theta_5) \cos(\theta_2) \cos(\theta_3),$
$\sin(\theta_6) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \cos(\theta_3)$
$+ \sin(\theta_6) \cos(\theta_4) \cos(\theta_5) \cos(\theta_2) \sin(\theta_3) - \sin(\theta_6) \sin(\theta_5) \sin(\theta_2) \sin(\theta_3)$
$+ \sin(\theta_6) \sin(\theta_5) \cos(\theta_2) \cos(\theta_3) + \sin(\theta_4) \cos(\theta_6) \sin(\theta_2) \cos(\theta_3)$
$+ \sin(\theta_4) \cos(\theta_6) \cos(\theta_2) \sin(\theta_3),$
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$- 0.4257792917 \cos(\theta_6) \cos(\theta_4) \cos(\theta_5) \cos(\theta_2) \sin(\theta_3)$
$+ 0.4257792917 \cos(\theta_6) \sin(\theta_5) \sin(\theta_2) \sin(\theta_3)$
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$+ 0.9048270524 \cos(\theta_5) \cos(\theta_2) \cos(\theta_3),$
$0.07850000000 \cos(\theta_6) \cos(\theta_4) \cos(\theta_5) \sin(\theta_2) \cos(\theta_3)$
$+ 0.07850000000 \cos(\theta_6) \cos(\theta_4) \cos(\theta_5) \cos(\theta_2) \sin(\theta_3)$
$- 0.07850000000 \cos(\theta_6) \sin(\theta_5) \sin(\theta_2) \sin(\theta_3)$
$+ 0.07850000000 \cos(\theta_6) \sin(\theta_5) \cos(\theta_2) \cos(\theta_3)$
$- 0.07850000000 \sin(\theta_4) \sin(\theta_6) \sin(\theta_2) \cos(\theta_3)$
$- 0.07850000000 \sin(\theta_4) \sin(\theta_6) \cos(\theta_2) \sin(\theta_3)$
$- 0.07850000000 \sin(\theta_4) \sin(\theta_6) \cos(\theta_2) \sin(\theta_3)$
$+ 0.5250000000 - 0.2540000000 \cos(\theta_4) \sin(\theta_5) \sin(\theta_2) \cos(\theta_3)$
$- 0.2540000000 \cos(\theta_4) \sin(\theta_5) \cos(\theta_2) \sin(\theta_3) - 0.2540000000 \cos(\theta_5) \sin(\theta_2) \sin(\theta_3)$
$+ 0.2540000000 \cos(\theta_5) \cos(\theta_2) \cos(\theta_3) - 0.7400000000 \sin(\theta_2) \sin(\theta_3)$
$+ 0.7400000000 \cos(\theta_2) \cos(\theta_3) - 0.1000000000 \sin(\theta_2) \cos(\theta_3)$
$- 0.1000000000 \cos(\theta_2) \sin(\theta_3) - 0.7700000000 \sin(\theta_2)], [0., 0., 0., 1.]$

2.2 VERIFICATION

In order to verify the direct kinematics calculated for the manipulator, we set all joint angles to zero and compare the graphical result of the simulator in Fig. 1 to the T calculated.

$$T_{\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0} = \begin{bmatrix} 0.9048270524 & 0 & 0.4257792917 & 0.9415000000 \\ 0. & 1 & 0. & 0. \\ -.4257792917 & 0 & 0.9048270524 & 1.519000000 \\ 0. & 0. & 0. & 1. \end{bmatrix}$$

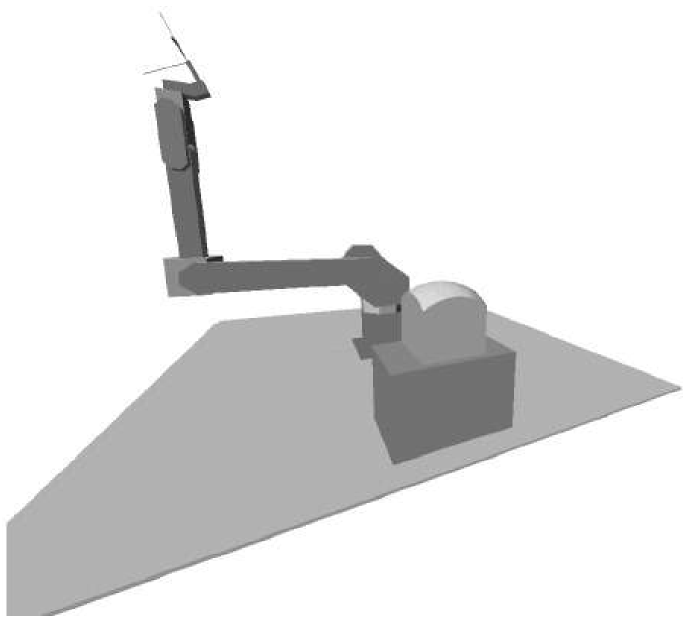

Figure 1. The simulated pose of the manipulator with all joint angles equal to zero

We can see in Fig. 1 that the tool orientation corresponds to the T calculated. Therefore we assume the direct kinematics is correct.

3 INVERSE KINEMATICS

3.1 COORDINATES OF THE EDGE POINT

In order to determine the pose of the manipulator for the task of deburring, we need the coordinates of the edge point. We know the spherical block has the dimensions of (0.25m x 0.25 m) in x and y, a radius r of 0.25 m with a center at C (0 ,1, 0.3). Thus the coordinates of the edge point P with x negative and y maximal can be obtained in the following steps.

From the information given it can be seen easily that $P_x = -0.125$ and $P_y = 1.125$

P_z can then be determined by inserting P_x and P_y in the sphere equation

$$P_z = C_z + sqrt\left(r^2 - \left(P_x - C_x\right)^2 - \left(P_y - C_y\right)^2\right) = 0.4767766953 \tag{2}$$

Thereafter we need to assign a coordinate system to the edged point P.

3. 2 COORDINATE SYSTEM AT POINT P

We want the z axis (v_z) to be normal to the surface of the sphere in point P pointing toward the sphere center C. Thus:

$$v_z = \frac{(C - P)}{r}$$

$$\begin{bmatrix} 0.500000000000000000 \\ -.500000000000000000 \\ -.707106781000000018 \end{bmatrix} \tag{3}$$

There are many solutions to obtain unit vectors in the x and y direction. We choose the following:

$$v_x = v_{x_0} \times v_z$$

$$v_x := \frac{v_x}{sqrt\left(v_x . v_x\right)} =$$

$$\begin{bmatrix} 0. \\ 0.816496581151255008 \\ -.577350269500000014 \end{bmatrix}$$

where v_{x_0} is the unit vector in x-direction.

(4)

$$v_y = v_z \times v_x$$

$$v_y = \frac{v_y}{sqrt(v_y \cdot v_y)}$$

$$\begin{bmatrix} 0.866025404000000054 \\ 0.288675134800000010 \\ 0.408248290600000008 \end{bmatrix} \tag{5}$$

3.2 TRANSFORMATION MATRIX A_b

The transformation matrix A_b describes the position required for the end-effector base to let the tool point at P.

$$A_b = \begin{bmatrix} v_{x_x} & v_{y_x} & v_{z_x} & P_x \\ v_{x_y} & v_{y_y} & v_{z_y} & P_y \\ v_{x_z} & v_{y_z} & v_{z_z} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \left(A_{tool}\right)^{-1}$$

$$\begin{bmatrix} 0.2128896458 & 0.8660254040 & 0.4524135262 & -.1779598458 \\ 0.5258985491 & 0.2886751348 & -.8000608621 & 1.289492409 \\ -.8234735670 & 0.4082482906 & -.3939855555 & 0.4728077958 \\ 0. & 0. & 0. & 1. \end{bmatrix} \tag{6}$$

3.3 CALCULATIONS OF THE JOINT ANGLES θ FOR THE EDGE POINT

As our robot is a decoupled manipulator (a manipulator whose last three joints have intersecting axes), the orientation problem is solved independently from the positioning problem. To solve the inverse kinematics problem (IKP), we follow the solution for decoupled manipulators described in the book of J. Angeles [4]. In the following we will thus just present the results of the coefficients without the formulas.

3.3.1 THE POSITIONING PROBLEM

The first step to solve the positioning problem is to calculate the intersecting point of the last three axes:

$$C_{wrist}=\begin{bmatrix} -0.2232 \\ 1.3695 \\ 0.5122 \end{bmatrix} \tag{7}$$

Hence with C_{wrist} known the coefficients are computed as:

$$A=-0.0670 \tag{8}$$

$$B=0.4108 \tag{9}$$

$$C=0.1540 \tag{10}$$

$$D=1.1396 \tag{11}$$

$$E=-0.7975 \tag{12}$$

$$F=-1.3695 \tag{13}$$

$$G=-0.2232 \tag{14}$$

$$H=I=J=0 \tag{15}$$

$$K=0.0237 \tag{16}$$

$$L=1.2987 \tag{17}$$

$$M=0.3510 \tag{18}$$

$$N=-0.2456 \tag{19}$$

$$P=-1.8177 \tag{20}$$

$$Q=0.4835 \tag{21}$$

$$R=0.7529 \tag{22}$$

$$S=-4.3374 \tag{23}$$

$$T=6.1144 \tag{24}$$

$$U=-2.9334 \tag{25}$$

V=0.2616

(26)

Thereafter, θ_3 is found solving the roots of the following polynomial:

$$R\tau_3^4 + S\tau_3^3 + T\tau_3^2 + U\tau_3 + V = 0$$

where $\tau_3=\tan\left(\frac{\theta_3}{2}\right)$

(27)

gives

θ_3 =2.645, 1.4364+0.2565i, 1.4364-0.2565i, 0.2274

(28)

with only two real solutions

$\theta_{3\text{real}}$ = 2.645, 0.2274

(29)

Then, we can find the corresponding θ_1 for each real θ_3 (For more details in the calculations please refer to [4])

θ_1 =1.7324, 1.7324

(30)

For θ_2 we need to compute additional coefficients:

A_{11} = 1.0343, 1.0343

(31)

A_{12} =-0.6984, 0.6984

(32)

Δ_2 =1.5575, 1.5575

(33)

providing the values of

θ_2 =-0.5836, 0.6043

(34)

With having the values of θ_1 , θ_2 and θ_3 the positioning problem is solved and we can continue with the orientation problem.

3.3.1 THE ORIENTATION PROBLEM

The orientation problem consist of finding θ_4 , θ_5 and θ_6 with:

$$\mathbf{R}=\boldsymbol{Q}_3^T\boldsymbol{Q}_2^T\boldsymbol{Q}_1^T\boldsymbol{Q} = \boldsymbol{Q}_4\boldsymbol{Q}_5\boldsymbol{Q}_6$$

where Q_i are the orientation parts of T_i (See Eq. 1)

(35)

with the information from the positioning problem we can calculate R:

$$\mathbf{R}_1 = \begin{bmatrix} 0.4975 & -0.4287 & 0.7542 \\ -0.2947 & -0.9012 & -0.3178 \\ 0.8159 & -0.0641 & -0.5747 \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} 0.9352 & 0.2036 & 0.2898 \\ -0.2947 & -0.9012 & -0.3178 \\ -0.1964 & 0.3826 & -0.9028 \end{bmatrix} \tag{35.1}$$

as $\mathbf{R}$ is equal to:

$$\mathbf{R} = \begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & c4s5 \\ s4c5c6 + c4c6 & -s4c5s6 + c4c6_2 & s4s51 \\ -s5c6 & s5s6_2 & c5 \end{bmatrix}$$

with $ci=\cos(\theta_i)$ and $si = \sin(\theta_i)$ (35.2)

when Eq.(35.2) is compared with equations. (35.1) the remaining angles can be found.

$$\theta_4 = -0.3988,\ -2.3100 \tag{36}$$

$$\theta_5 = \pm 2.183,\ \pm 2.6970 \tag{37}$$

$$\theta_6 = -3.0631,\ 1.0966 \tag{38}$$

Hence the solutions for the manipulator pose are the following:

Solution	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	1.7324	-0.5836	2.645	-0.3988	2.183	3.0631
2	1.7324	-0.5836	2.645	-0.3988	-2.183	3.0631
3	1.7324	0.6043	0.2274	-2.3100	2.697	1.0966
4	1.7324	0.6043	0.2274	-2.3100	-2.697	1.0966

4 VERIFICATION

To verify the solutions the results are given as input to the robot simulator [2] which gives the following graphical results:

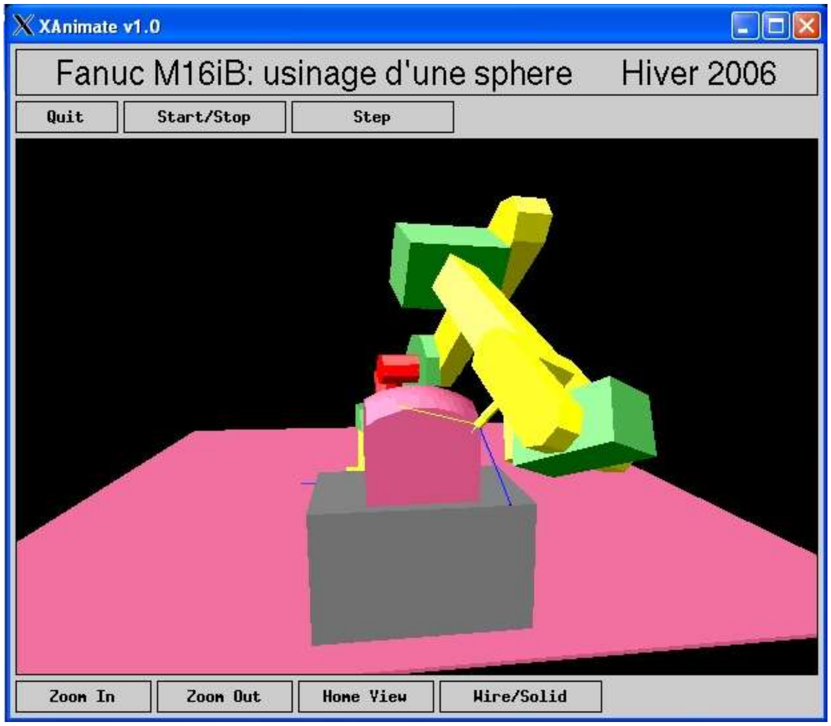

Figure 2. Simulated pose for solution 1

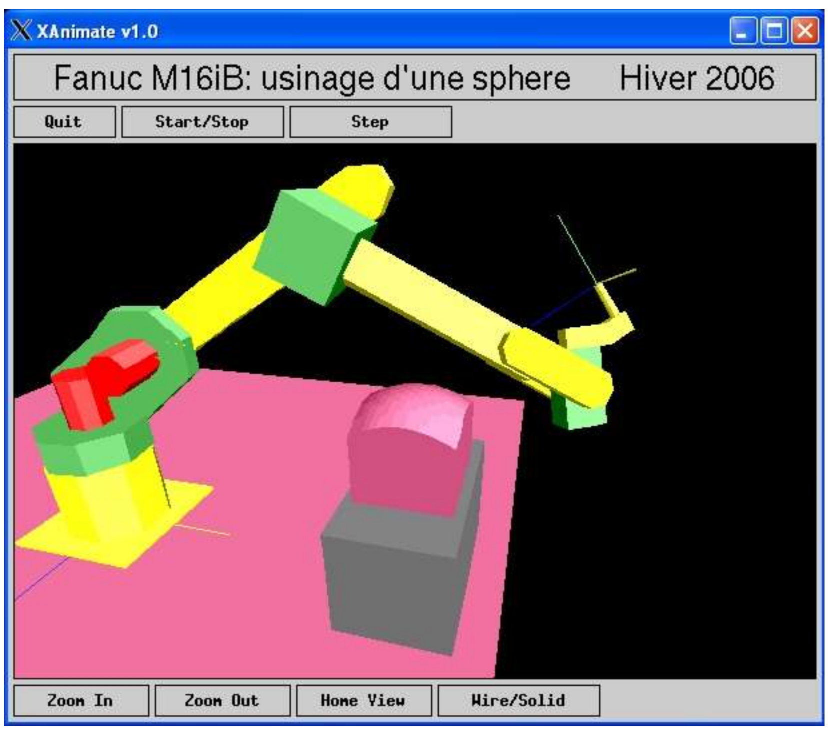

Figure 3. Simulated pose for solution 2

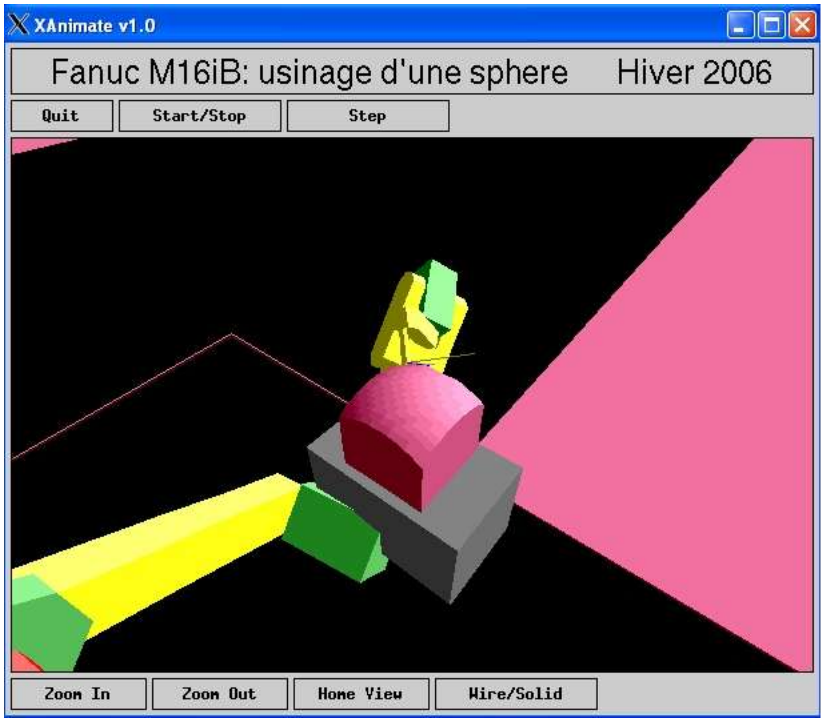

Figure 4. Simulated pose for solution 3

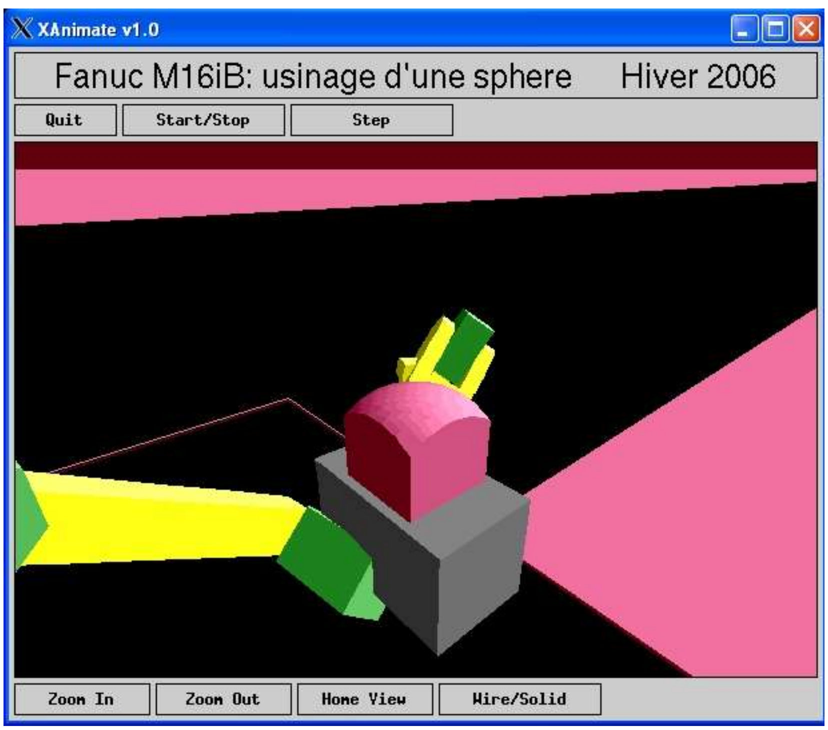

Figure 5. Simulated pose for solution 4

It can be seen in Fig. 3 and 5 that the solutions 2 and 4 do not point to the edge point. Solution 3 (see Fig. 4) is considered as a theoretical one, as the arm of the robot would penetrate the support block of the sphere block. The only solution that seems to be valid is solution 1, as seen in Fig. 2. Even then it is not absolutely certain that the robot can take this pose, as joint number 5 is bended quite extremely.

5 DISCUSSION

For the task of deburring the edge of a spherical block, we have found four possible solutions with the inverse kinematics. Unfortunately the verification with the simulator showed that only one of our four solutions is practical. Solutions 2 and 4 that do not point at the edge point at all, are completely wrong and only given as mirror solutions of the trigonometric equations. Solution 1 and 4 are in theory correct but as said before, solution 4 is not physically possible for the manipulator because of physical constraints.

Solution 1 seems to be the only practical solution. If however solution 1 would turn out not to be achievable with the manipulator, the calculations would need to be redone with a different orientation of the tool pointing at the edge point. (There are infinite orientation possibilities as the tool can turn around its own axis without affecting the deburring process.)

6 REFERENCES

[1] FANUC Robotics America Inc. www.fanucrobotics.com

[2] Luc Baron, École Polytechnique de Montréal, Simulation Software of FANUC robot

[3] Luc Baron, École Polytechnique de Montréal, MEC6503 Task Project1

[4] Angeles, J., Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer, New York, 2nd edition, 2003.