The optimum Kinematic Design of a Spherical Three-Degree-of-Freedom Parallel Manipulator – How to obtain the Jacobian

Stefan Bracher

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In their fundamental paper « The Optimum Kinematic Design of a Spherical Three-Degree-of-Freedom Parallel Manipulator » [1] C. Gosselin and J.Angeles presented the Jacobian of the studied cinematic, which has been cited by many other researchers.

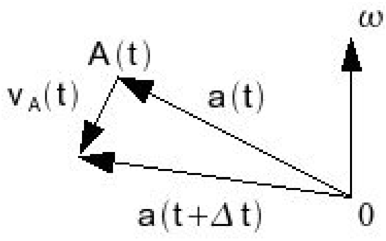
However, how the Jacobian was obtained is not written in detail in the paper. The missing lines shall be given in the following.

In [1] it is suggested to obtain the Jacobian by differentiating the Enclosure Equation:

$$w_i * v_i = \cos(\alpha_2) \tag{7}$$

what leads to:

$$\mathring{w}_i * v_i + w_i * \mathring{v}_i = 0 \tag{17}$$



$a(t+\Delta t) = a(t) + v_A(t)$ *for* $\Delta t \to 0$

The derivation of a (unitary) vector going through the origin of the coordinate system, undergoing a rotation around the origin itself, can be seen as the speed of the end-point of this vector.

w, v , u and ω are intersecting in the origin (the middle of the sphere). Assuming that we know the rotational vector ω , the derivative of v_i can be written as:

$$\mathring{v}_i = \omega \times v_i \tag{a1}$$

and the derivative of w_i becomes:

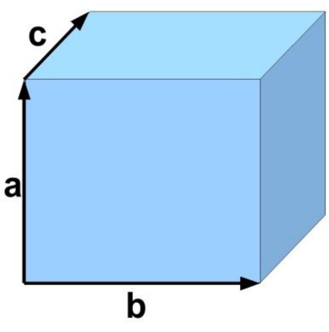
$$\mathring{w}_i = \mathring{\Theta}_i * u_i \times w_i \tag{a2}$$

Inserting (a1) and (a2) in (17) gives:

$$\mathring{\Theta}_i * (u_i \times w_i) * v_i + w_i * (\omega \times v_i) = 0 \tag{a3}$$

Rearranging of (a3) under consideration of $c * (a \times b) = (a \times b) * c = -(c \times b) * a$ gives:

$$\mathring{\Theta}_i = \frac{(w_i \times v_i) * \omega}{(u_i \times w_i) * v_i} \tag{a4}$$



$(a \times b) * c = -(c \times b) * a$

Which is* equation (19) of [1].

*In the paper [1], a typing error, exchanging a cross product with a dot product must have happened.

The solution of the i-th row of the Jacobian is thus as stated in [1]:

$$\mathbf{J}_i = \frac{(\mathbf{w}_i \times \mathbf{v}_i)^T}{(\mathbf{u}_i \times \mathbf{w}_i) * \mathbf{v}_i}$$

References

[1] C.Gosselin, J.Angeles, « The optimum kinematic design of a spherical three-degee-of-freedom parallel manipulator », *Journal of Mechanisms, Transmissions and Automation in Design Vol. 111*, June 1989