Rotating Table with Parallel Kinematic Featuring a Planar Joint

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- **Outline**
  - Introduction
  - The kinematic design
  - Kinematics
  - Jacobian Matrix
  - Singularities
  - Workspace
  - Conclusion

- **Introduction**
  - 3 DOF milling machines
  - Additional DOF with Rotating Tables
  - Parallel Kinematics to reduce inertia

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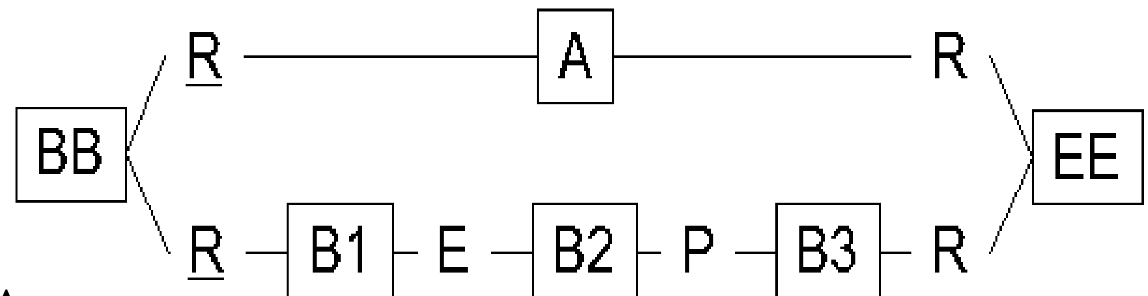
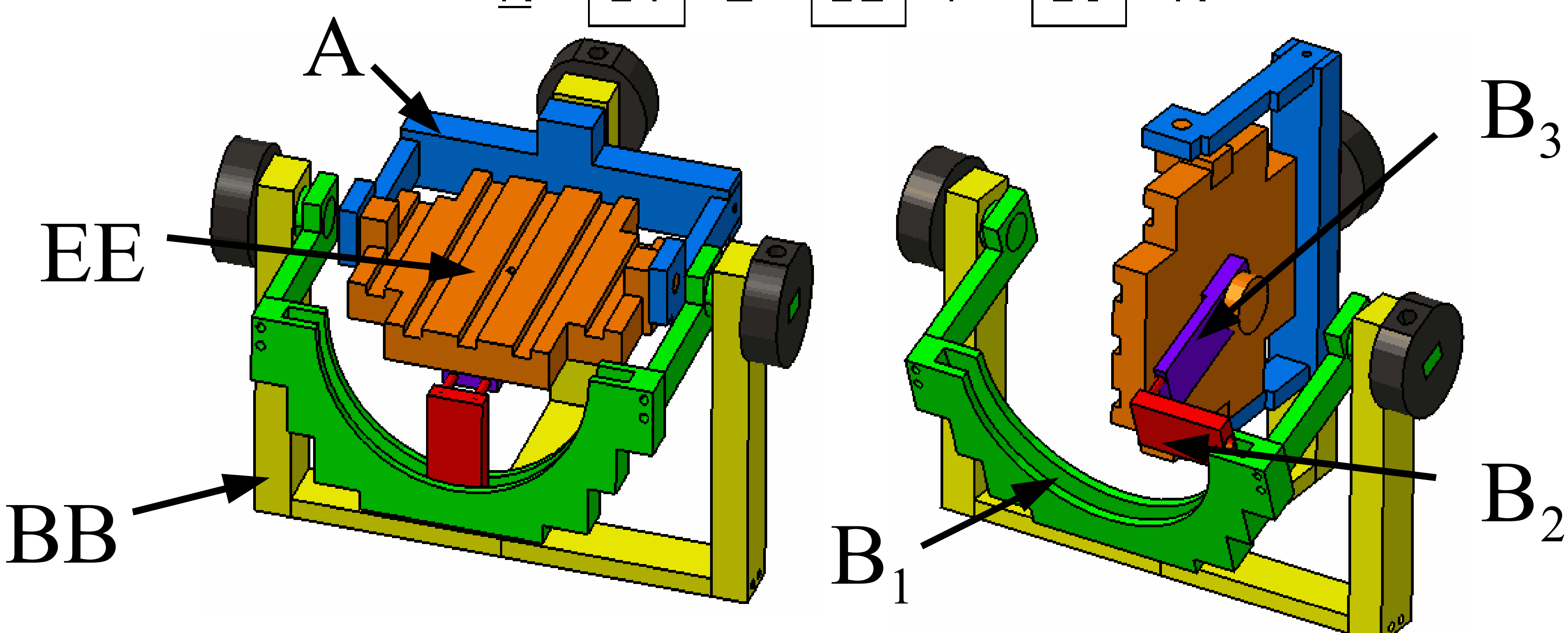
- **Kinematic design**





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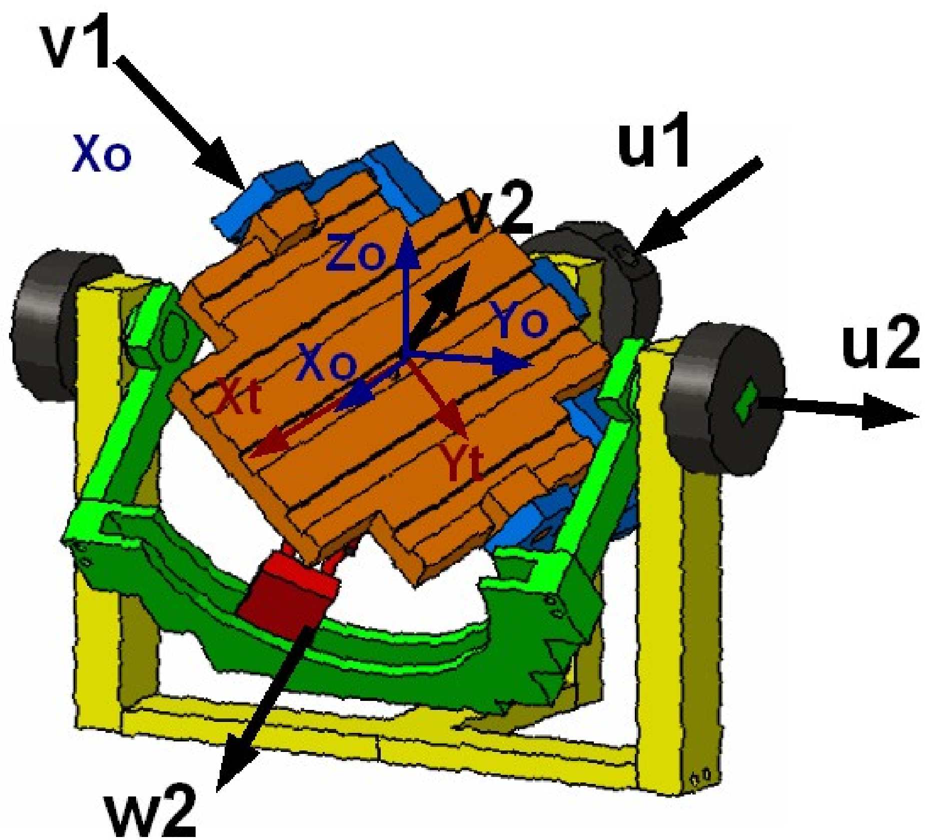
- **Kinematic design**



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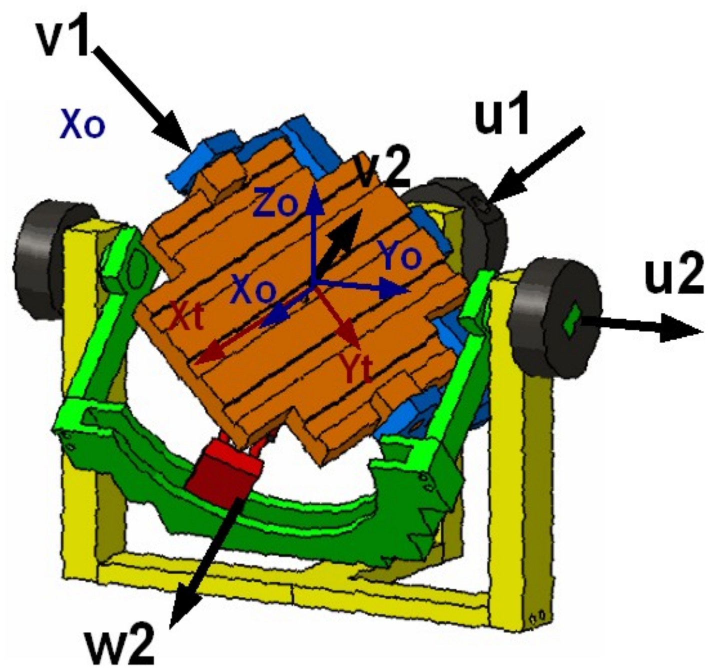
- **Kinematics**
  - **Inverse Kinematics**
    - Simple



$$\theta_1 = \text{atan2}\left(\frac{{}_0\mathbf{v}_1(3)}{{}_0\mathbf{v}_1(2)}\right)$$

$$\theta_2 = \text{atan2}\left(\frac{{}_0\mathbf{v}_2(1)}{{}_0\mathbf{v}_2(3)}\right)$$

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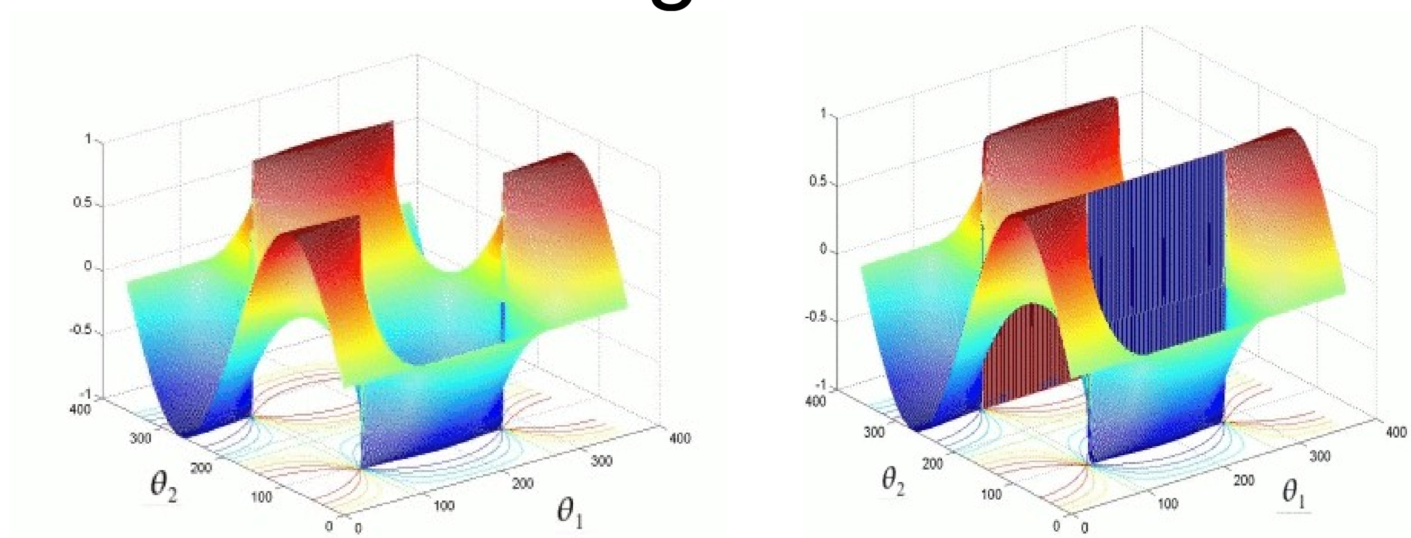
- **Kinematics**
  - **Direct Kinematics**
    - Multiple Solutions

→ Algorithm



Different values for v2(1) if Theta1 is moved first (left) or Theta2 is moved first (right)

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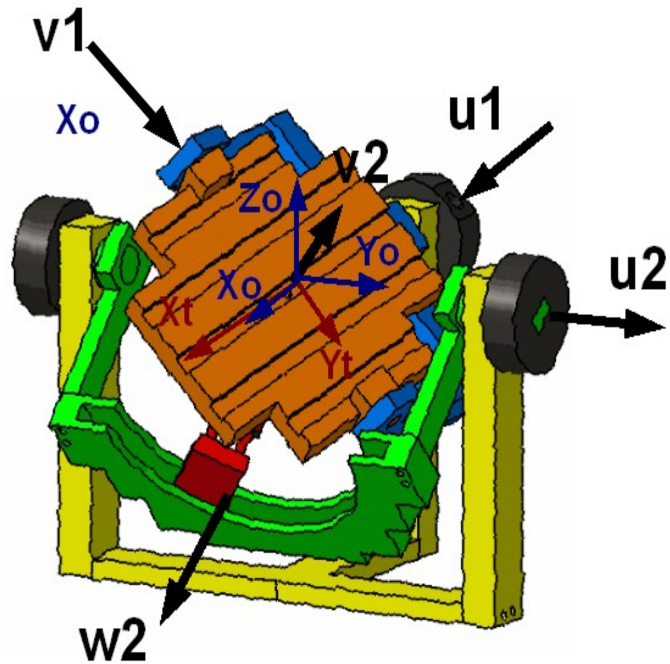
- **Jacobian Matrix**
  - Closure Equations



$$\mathbf{v}_2 \perp \mathbf{w}_2 \Rightarrow \mathbf{v}_2 \cdot \mathbf{w}_2 = 0$$

$$\mathbf{v}_1 \perp \mathbf{v}_2 \Rightarrow \mathbf{v}_1 \cdot \mathbf{v}_2 = 0$$

$$\mathbf{v}_1 \perp \mathbf{u}_1 \Rightarrow \mathbf{v}_1 \cdot \mathbf{u}_1 = 0$$

$$\begin{bmatrix} 0 & \mathbf{u}_2 \times \mathbf{w}_2 \cdot \mathbf{v}_2 \\ \mathbf{u}_1 \times \mathbf{v}_1 \cdot \mathbf{v}_2 & 0 \\ 0 & 0 \end{bmatrix} \dot{\boldsymbol{\theta}} = \begin{bmatrix} (\mathbf{w}_2 \times \mathbf{v}_2)^T \\ (\mathbf{v}_1 \times \mathbf{v}_2)^T \\ (\mathbf{u}_1 \times \mathbf{v}_1)^T \end{bmatrix} \boldsymbol{\omega}$$

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- **Jacobian Matrix**

$$\mathbf{A}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\mathbf{B}\,\boldsymbol{\omega}:\begin{bmatrix}0 & \mathbf{u}_2\times\mathbf{w}_2\cdot\mathbf{v}_2\\ \mathbf{u}_1\times\mathbf{v}_1\cdot\mathbf{v}_2 & 0\\ 0 & 0\end{bmatrix}\dot{\boldsymbol{\theta}}=\begin{bmatrix}(\mathbf{w}_2\times\mathbf{v}_2)^{\mathrm{T}}\\ (\mathbf{v}_1\times\mathbf{v}_2)^{\mathrm{T}}\\ (\mathbf{u}_1\times\mathbf{v}_1)^{\mathrm{T}}\end{bmatrix}\boldsymbol{\omega}$$

$$\mathbf{J}_{\theta\rightarrow\omega}=\mathbf{B}^{-1}\mathbf{A}$$

$$\mathbf{J}_{\theta\rightarrow\omega}\,\dot{\boldsymbol{\theta}}=\boldsymbol{\omega}$$

$$\mathbf{J}_{\omega\rightarrow\theta}=\mathbf{A}^{-1}\mathbf{B}$$

$$\dot{\boldsymbol{\theta}}=\mathbf{J}_{\omega\rightarrow\theta}\,\boldsymbol{\omega}$$

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- **Jacobian Matrix**

$$\mathbf{A}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}=\mathbf{B}\boldsymbol{\omega}:\begin{bmatrix}0 & \mathbf{u}_2\times\mathbf{w}_2\cdot\mathbf{v}_2\\ \mathbf{u}_1\times\mathbf{v}_1\cdot\mathbf{v}_2 & 0\\ 0 & 0\end{bmatrix}\dot{\boldsymbol{\theta}}=\begin{bmatrix}(\mathbf{w}_2\times\mathbf{v}_2)^{\mathrm{T}}\\ (\mathbf{v}_1\times\mathbf{v}_2)^{\mathrm{T}}\\ (\mathbf{u}_1\times\mathbf{v}_1)^{\mathrm{T}}\end{bmatrix}\boldsymbol{\omega}$$

$$\mathbf{J}_{\theta\to\omega}=\mathbf{B}^{-1}\mathbf{A}$$

$$\mathbf{J}_{\theta\to\omega}\dot{\boldsymbol{\theta}}=\boldsymbol{\omega}$$

$$\mathbf{J}_{\omega\to\theta}=\mathbf{A}^{-1}\mathbf{B}$$

$$\dot{\boldsymbol{\theta}}=\mathbf{J}_{\omega\to\theta}\boldsymbol{\omega}$$

→ $\mathbf{u}_1\times\mathbf{v}_1\cdot\boldsymbol{\omega}=0$

→ $\boldsymbol{\omega}\perp(\mathbf{u}_1\times\mathbf{v}_1)$

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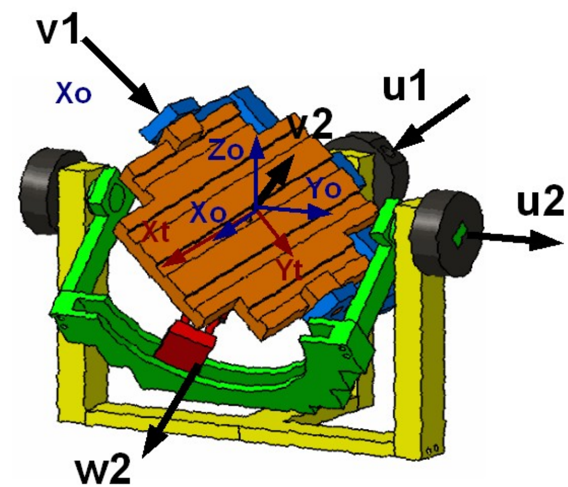
• Jacobian Matrix

$$\boldsymbol{\omega} \perp (\mathbf{u}_1 \times \mathbf{v}_1)$$



$$\mathbf{A}^x \dot{\boldsymbol{\theta}} = \mathbf{B}^x \boldsymbol{\omega}^x : \quad \begin{bmatrix} 0 & \mathbf{u}_2 \times \mathbf{w}_2 \cdot \mathbf{v}_2 \\ \mathbf{u}_1 \times \mathbf{v}_1 \cdot \mathbf{v}_2 & 0 \end{bmatrix} \dot{\boldsymbol{\theta}} = \begin{bmatrix} (\mathbf{w}_2 \times \mathbf{v}_2)^T \\ (\mathbf{v}_1 \times \mathbf{v}_2)^T \end{bmatrix} \cdot \vec{\boldsymbol{\omega}}^x$$

$$\mathbf{J}^x_{\omega \to \theta} = \mathbf{A}^{x-1} \mathbf{B}^x$$

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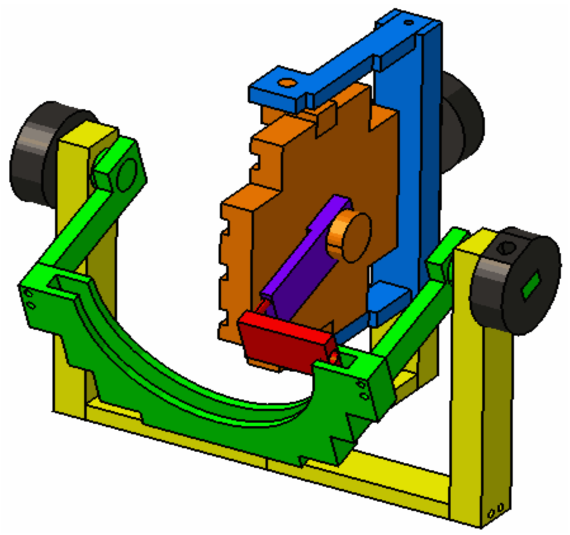
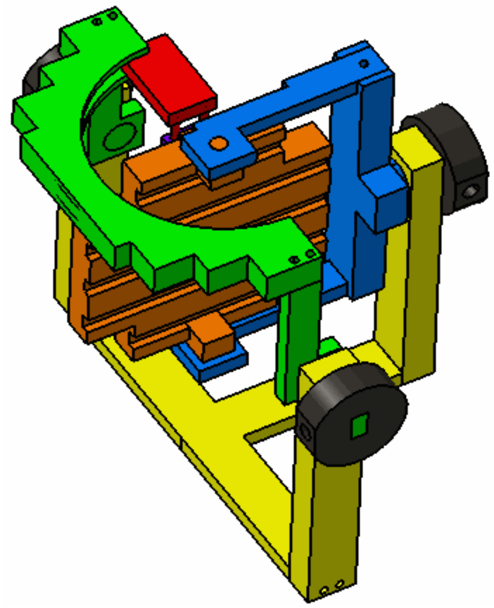
- **Singularities**



$\mathbf{v}_1 = \pm \mathbf{z}_0, \mathbf{v}_2 = \pm \mathbf{u}_2$



$\mathbf{w}_2 = \pm \mathbf{v}_1 = \pm \mathbf{z}_0$

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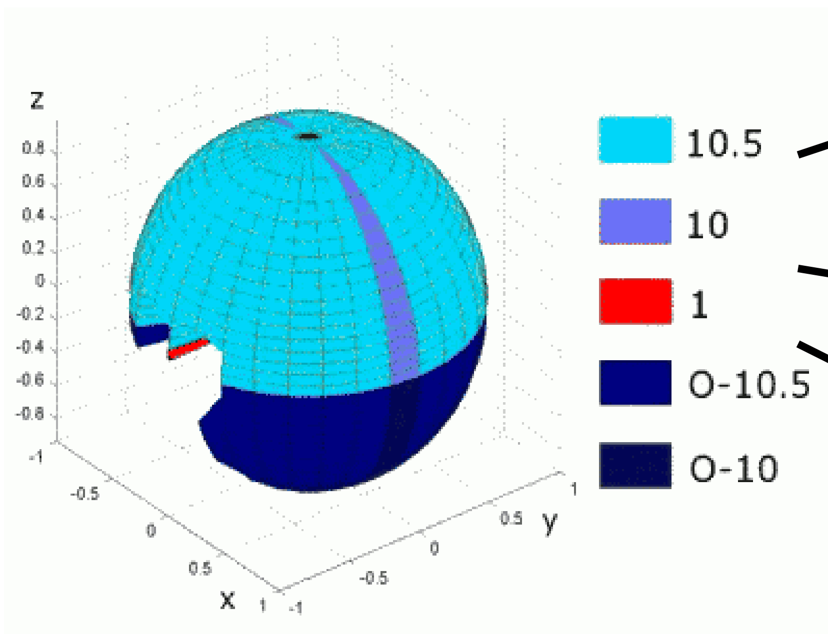
- **Workspace**



Mode Theta1 before Theta2

$$\mathbf{J}_{\theta\to\omega} = \begin{bmatrix} \mathbf{a}' \\ \mathbf{b}' \\ \mathbf{c}' \end{bmatrix}$$

$$\mathbf{J}_{\theta\to\omega} = \begin{bmatrix} \mathbf{a}' \\ \mathbf{b}' \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{J}_{\theta\to\omega} = \begin{bmatrix} \mathrm{a} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{J}_{\theta\to\omega} = \begin{bmatrix} 0 & \mathrm{a} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

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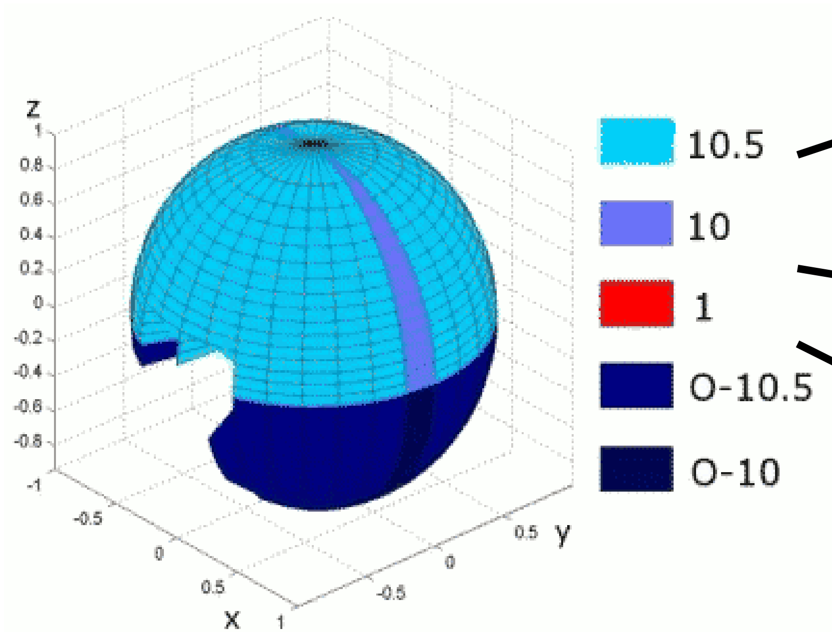
- **Workspace**



Mode Theta2 before Theta1

$$\mathbf{J}_{\theta\to\omega}=\begin{bmatrix}\mathbf{a}'\\ \mathbf{b}'\\ \mathbf{c}'\end{bmatrix}$$

$$\mathbf{J}_{\theta\to\omega}=\begin{bmatrix}\mathbf{a}'\\ \mathbf{b}'\\ \mathbf{0}\end{bmatrix}$$

$$\mathbf{J}_{\theta\to\omega}=\begin{bmatrix}\mathrm{a} & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},\mathbf{J}_{\theta\to\omega}=\begin{bmatrix}0 & \mathrm{a}\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

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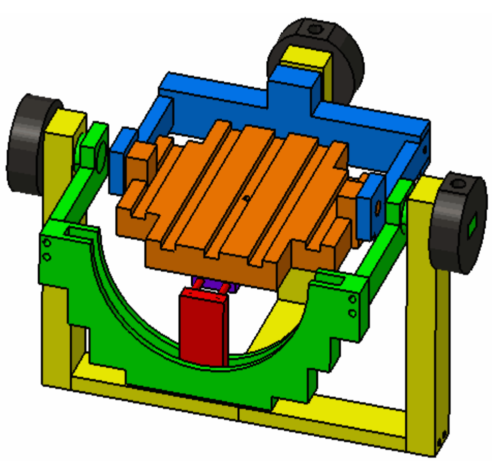
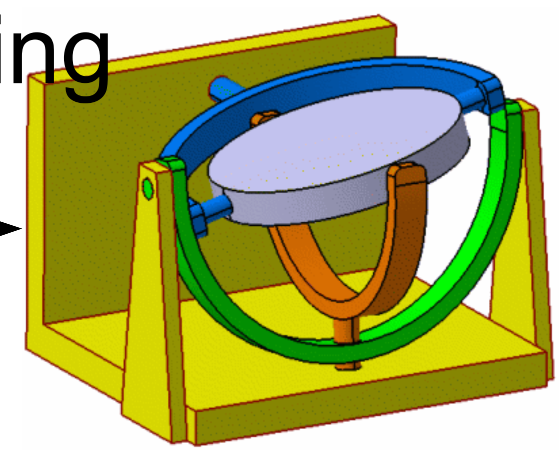
- **Conclusion**
  - **Advantages of the planar joint:**
    - Free access from the front when rotation center is above the surface
    - Larger Workspace
    - Higher manufacturing tolerances





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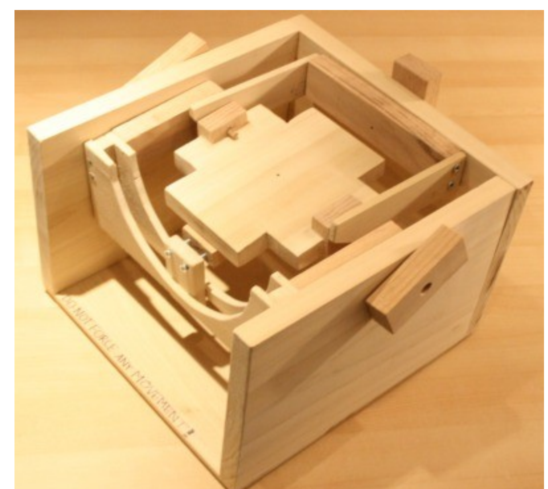
- **Conclusion**
  - Singularities at obstructed orientations
  - Almost +-90° in two directions
  - To be done: Controller



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- **Direct Kinematics**

$$ {}_{0}\mathbf{v}_1 = {}_{0}\begin{pmatrix} 0 \\ \cos(\theta_1) \\ \sin(\theta_1) \end{pmatrix} $$

$$ \mathbf{v}_2 \perp \mathbf{v}_1 \rightarrow \mathbf{v}_2 \cdot \mathbf{v}_1 = 0 $$
$$ \mathbf{v}_2 \perp \mathbf{w}_2 \rightarrow \mathbf{v}_2 \cdot \mathbf{w}_2 = 0 $$
$$ |\mathbf{v}_2| = 1 $$

for ${}_{0}\mathbf{v}_2(3) \neq 0$:

$$ {}_{0}\mathbf{v}_2 = {}_{0}\begin{pmatrix} \tan(\theta_2)\, {}_{0}v_2(3) \\ -\tan(\theta_1)\, {}_{0}v_2(3) \\ {}_{0}v_2(3) \end{pmatrix} $$

with ${}_{0}v_2(3) = \pm\sqrt{\dfrac{1}{1+\tan(\theta_1)^2+\tan(\theta_2)^2}}$

for ${}_{0}\mathbf{v}_2(3) = 0$

multiple geometrical solutions

$$ \mathbf{x}_t = \mathbf{z}_t \times \mathbf{y}_t, \quad \mathbf{y}_t = \mathbf{v}_1, \quad \mathbf{z}_t = \mathbf{v}_2 $$

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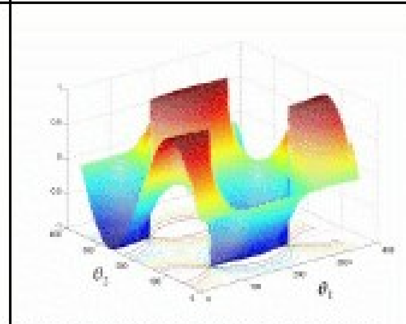
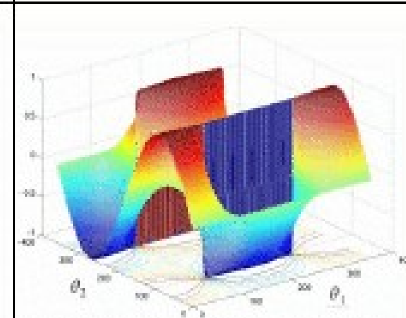
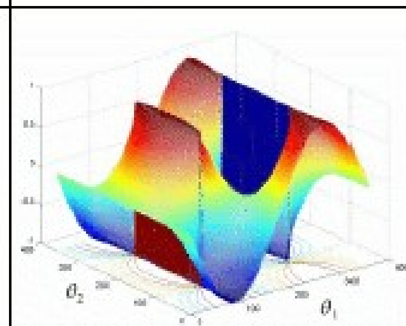
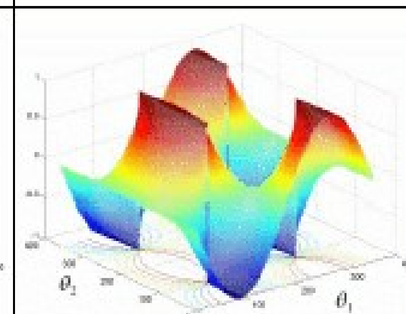
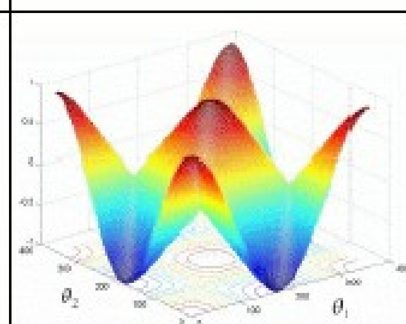
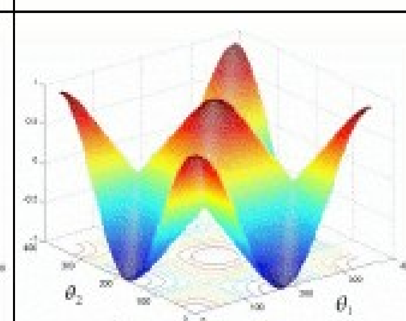
Additional Slide

	θ_1 en premier	θ_2 en premier
$\mathbf{v}_2(1)$	 Figure 42: v2(1) en mode theta1 en premier	 Figure 43: v2(1) en mode theta2 en premier
$\mathbf{v}_2(2)$	 Figure 44: v2(2) en mode theta1 en premier	 Figure 45: v2(2) en mode theta2 en premier
$\mathbf{v}_2(3)$	 Figure 46: v2(3) en mode theta1 en premier	 Figure 47: v2(3) en mode theta2 en premier

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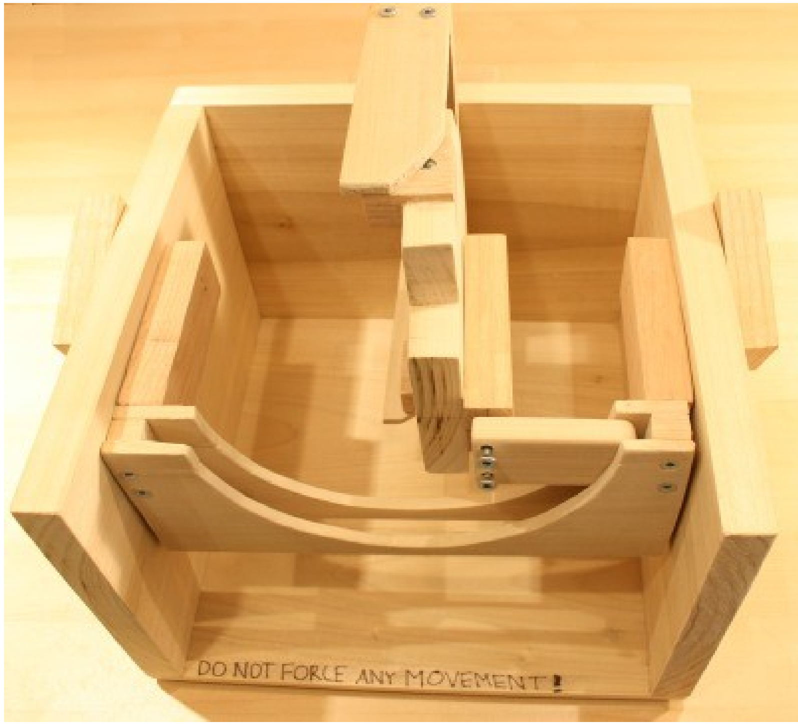
Additional Slide



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Additional Slide

```
%   Title:         drill_orientation
%   Author:       Stefan Bracher (http://stefan.bracher.info)
%   Date:           07.09.2006
%   Description:   Finds the table position to a drill orientation
%                        Returns 0 when not possible
%   Dependencies: none
%
%   Syntax:            [T1, T2]=drill_orientation(drill)
%
%   Input:             drill:      drill vector
%
%   Output:            T1  : Corresponding table orientation mode
%                                Theta2->Theta1
%                            T2  : Corresponding table orientation mode
%                                Theta1->Theta2

function [T1, T2]=drill_orientation(drill)
T=[1 0 0 0; 0 1 0 0; 0 0 1 0; 0 0 0 1];            % Using the horizontal orientation
threshold=1e-6;                                   % what is regarded as zero


%%%%%%MODE THETA2->THETA1 %%%%%%%%%%%%
%Turn around y
roty=-atan2(drill(1),drill(3));
if ((drill(3)==0)&&(drill(1)==0))

        roty=sign(drill(2))*pi/2;
end
roty_deg=roty*360/(2*pi);
Roty=[cos(roty) 0 sin(roty) 0; 0 1 0 0; -sin(roty) 0 cos(roty) 0; 0 0 0 1];
T1=Roty*T;
drill_i=Roty(1:3,1:3)*drill;


%Turn around x0
rotx=atan2(drill_i(2),drill_i(3));
rotx_deg=rotx*360/(2*pi);
Rotx=[1 0 0 0; 0 cos(rotx) -sin(rotx) 0; 0 sin(rotx) cos(rotx) 0; 0 0 0 1];
T1=Rotx*T1;
drill_f=Rotx(1:3,1:3)*drill_i;
%Check:
if ((abs(1-drill_f(3))<threshold)&&(abs(drill_f(1))<threshold)&&(abs(drill_f(2))<threshold))
%ok
else
T1=0;
end
```

```
%%%%%%%%%%%%MODE Theta1->Theta2%%%%%%%%%%%%%%%%

% turn around x_0
yt=[0 1 0]';
R=[0 drill(2) 0]';
r=norm(R-drill);
rotx=sign(drill(2))*(pi/2-asin(r));
rotx_deg=rotx*360/(2*pi);
Rotx=[1 0 0 0; 0 cos(rotx) -sin(rotx) 0; 0 sin(rotx) cos(rotx) 0; 0 0 0 1];
T2=Rotx*T;
drill_i=Rotx(1:3,1:3)*drill;
R_i=Rotx(1:3,1:3)*R;
yt_i=Rotx(1:3,1:3)*yt;


%turn around yt
R_iZ=[0 0 1]'-R_i;
R_idrill_i=drill_i-R_i;
if (norm(R_idrill_i)==0)

        rotxy=0;



else                          rotxy=acos(dot(R_iZ,R_idrill_i)/(norm(R_iZ)*norm(R_idrill_i)));

end


if (sign(drill_i(1))>0)
rotxy=-sign(drill_i(1))*rotxy;
end


if (sign(drill_i(1))<0)
rotxy=-sign(drill_i(1))*rotxy;
end


rotxy_deg=

.....
```