Formulas and Constants for College Physics

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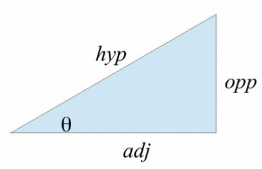
Math + Measurement

Quadratic Equations

Quadratic Equation $ax^2 + bx + c = 0 \rightarrow x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry



Soh-Cah-Toa

$\sin(\theta) = \frac{opp}{hyp}$

$\cos(\theta) = \frac{adj}{hyp}$

$\tan(\theta) = \frac{opp}{adj} = \frac{\sin(\theta)}{\cos(\theta)}$

Trigonometric Identitiess

Identities

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\frac{1}{\cos^2(\theta)} = \tan^2(\theta) + 1$

Vectors

Dot-Product $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z = a\, b \cos(\theta)$

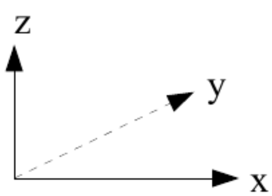
Cross-Product

$$\vec{a}\, x\, \vec{b} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} x \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = \begin{bmatrix} a_y b_z - b_y a_z \\ a_z b_x - b_z a_x \\ a_x b_y - b_x a_y \end{bmatrix}$$

$|\vec{a}\, x\, \vec{b}| = a\, b \sin(\theta)$

(Applies to Right-Hand Coordinate Systems only)

Right hand coordinate system



Addition/Subtraction Head-to-Tail method or by components

Logarithms

$\ln(x^S) = s\ln(x)$	$\ln(e^x) = x$	$\ln(xy) = \ln(x) + \ln(y)$	$\ln(x/y) = \ln(x) - \ln(y)$

Linear Alegebra

System of Equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots &= c_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots &= c_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \ldots &= c_3 \\ &\ldots \end{aligned}$$

$\rightarrow$ $\boldsymbol{A\,x = C \rightarrow x = A^{-1}C}$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots \\ a_{21} & a_{22} & a_{23} & \ldots \\ a_{31} & a_{32} & a_{33} & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \ldots \end{bmatrix}$$

Uncertainty

Absolute	$x = x_{avg} \pm \Delta x$
Relative	$x = x_{avg} \pm \frac{\Delta x}{x}$
Addition/Subtraction	$(x_{avg} \pm \Delta x) + (y_{avg} \pm \Delta y) = (x_{avg} + y_{avg}) \pm (\Delta x + \Delta y)$
Multiplication/ Division	$(x_{avg} \pm \Delta x) * (y_{avg} \pm \Delta y) = x_{avg} * y_{avg} (1 \pm [\frac{\Delta x}{x_{avg}} + \frac{\Delta y}{y_{avg}}])$
Min-Max Method	$x_{best} \pm \frac{x_{max} - x_{min}}{2}$

Uncertainty is obtained by:

- Estimation (at least, ½ the lowest increment)*
- Statistics (mean deviation, standard deviation) **

* Usually much more (uncertainty of method, object, tool and observer add up)

** typically more than 10 measurements are needed

Estimated Digit: The last written digit of a number is estimated. Only one written digit should be affected by the uncertainty

Example: *1.50 ± 0.04*

Excel / LibreOffice / OpenOffice

General

Typing formulas and equations	Click on a cell and type =.......	Fixing a cell reference	Add a $ in front of the row or column. Ex. C4

Statistics

Average	AVERAGE(Range)	Highest, Lowest Value	MIN(Range) , MAX(Range)
Mean Deviation (Uncertainty)	AVEDEV(Range)	Standard Deviation	STDEV(Range)

Slopes and Intercept

Slope	INDEX(LINEST(Y_Values,X_Values, TRUE,TRUE),1,1)	Intercept	INDEX(LINEST(Y_Values,X_Values, TRUE,TRUE),2,1)
Uncertainty of Slope	INDEX(LINEST(Y_Values,X_Values, TRUE,TRUE),2,1)	Uncertainty of Intercept	INDEX(LINEST(Y_Values,X_Values, TRUE,TRUE),2,2)

Calculator Hints

Equation

Model	**Sharp**		**Casio**
	EL-520X	EL-W516X	fx-991 ES
Quadratic Equation	Mode → 2 → 2	Mode → 6 → 2	
System of equations	Mode → 2 → 0 / 1	Mode → 6 → 0/1	Mode → 5 → 2

Wolfram Alpha - http://www.wolframalpha.com/

Equation

	Example
Quadratic Equation	3x^2 - 2x - 2= 0 → Enter
System of equations	3x+2y+7z=4, 3x+y=0, y+z=0 → Enter

Mechanics

1D Kinematics

Position	**s=x**, **s=y** or **s=z** (sign gives direction)
Displacement	$\Delta \mathbf{s} = \mathbf{s}_f - \mathbf{s}_i$
Average velocity	$\mathbf{v}_{av} = \frac{\mathbf{s}_f - \mathbf{s}_i}{\Delta t}$
Instantaneous velocity	$\mathbf{v} = \frac{d\,\mathbf{s}}{d\,t}$ (slope of position)
Average acceleration	$\mathbf{a}_{av} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t}$
Instantaneous acceleration	$\mathbf{a} = \frac{d\,\mathbf{v}}{d\,t}$ (slope of velocity)
Five equations for **CONSTANT** acceleration	$\Delta v = at$ $\Delta s = 1/2(v_i + v_f)t$ $\Delta s = v_i t + 1/2\,a\,t^2$ $\Delta s = v_f t - 1/2\,a\,t^2$ $v_f^2 = v_i^2 + 2a\,\Delta s$

Graphs

a-t (→ area / slope ←) v-t (→ area / slope ←) s-t

2D/3D Kinematics

Position	$\vec{r} = (x, y, z)$
Displacement	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i = (\Delta x, \Delta y, \Delta z)$
Average velocity	$\vec{v}_{av} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} = (\frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t})$
Instantaneous velocity	$\vec{v} = \frac{d\,\vec{r}}{d\,t} = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt})$
Average acceleration	$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = (\frac{\Delta v_x}{\Delta t}, \frac{\Delta v_y}{\Delta t}, \frac{\Delta v_z}{\Delta t})$
Instantaneous acceleration	$\vec{a} = \frac{d\,\vec{v}}{d\,t} = (\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt})$
Five equations for **CONSTANT** acceleration	$\vec{v}_f = \vec{v}_i + \vec{a}\,t$ $\vec{r}_f = \vec{r}_i + 1/2(\vec{v}_i + \vec{v}_f)t$ $\vec{r}_f = \vec{r}_i + \vec{v}_i t + 1/2\,\vec{a}\,t^2$ $\vec{r}_f = \vec{r}_i + \vec{v}_f t - 1/2\,\vec{a}\,t^2$ $v_{fxyz}^2 = v_{ixyz}^2 + 2a_{xyz}\,\Delta r_{xyz}$

Projectile Motion

Path	$\vec{r}(t) = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}\,t^2$
Trajectory for $\vec{r}_i = 0$	$y = \tan(\Theta_i)\,x - \frac{g\,x^2}{2(v_i \cos(\Theta_i))^2}$
Range for $\vec{r}_i = 0$	$\Delta x = \frac{v_i^2 \sin(2\Theta_i)}{g}$

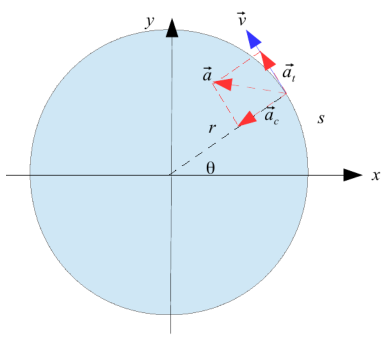
Circular motion

Angular position (Consider CCW of +x as positive)	$\theta = \frac{s}{r}$ [rad]
Angular displacement*	$\Delta\theta = \theta_f - \theta_i$ [rad]
Average angular velocity	$\boldsymbol{\omega}_{av} = \frac{\theta_f - \theta_i}{\Delta t}$ [rad/s]
Instantaneous velocity	$\boldsymbol{\omega} = \frac{d\,\boldsymbol{\theta}}{d\,t}$ (slope of angular position)
Average angular acceleration	$\boldsymbol{\alpha}_{av} = \frac{\boldsymbol{\omega}_f - \boldsymbol{\omega}_i}{\Delta t}$ [rad/s²]
Instantaneous acceleration	$\boldsymbol{\alpha} = \frac{d\,\boldsymbol{\omega}}{d\,t}$ (slope of velocity)
Five equations for **CONSTANT angular** acceleration	$\Delta\omega = \alpha\,t$ $\Delta\theta = 1/2(\omega_i + \omega_f)t$ $\Delta\theta = \omega_i t + 1/2\,\alpha\,t^2$ $\Delta\theta = \omega_f t - 1/2\,\alpha\,t^2$ $\omega_f^2 = \omega_i^2 + 2\alpha\,\Delta\theta$

Graphs

α-t (→ area / slope ←) ω-t (→ area / slope ←) Θ-t

*While angular velocity and angular acceleration are vectors, angular displacement ΔΘ is not. (Vector addition does not work)



Conversion to linear entities

Distance travelled	$s = \Delta\theta\,r$
Speed	$v = \omega\,r$
Tangential acc.	$a_t = \alpha\,r$
Centripetal acc.	$a_c = \frac{v^2}{r} = \omega^2 r$
Linear acceleration:	$\vec{a} = \vec{a}_t + \vec{a}_c$

Uniform Circular Motion (α=0)

Circumference: $2\pi r$

Frequency: f [Hz] f=1/T
(how many cycles per second)

Period: T (time to go around once)

Speed v = $\frac{2\pi r}{T} = \omega r$

Centripetal acc. $a_c = \frac{v^2}{r} = \omega^2 r$

Velocity $\vec{v} = -v\sin(\theta(t))\,\hat{i} + v\cos(\theta(t))\,\hat{j}$ $\theta(t) = \theta_i + \omega\,t$

Acceleration $\vec{a} = \dot{\vec{v}} = -a_c\cos(\theta(t))\,\hat{i} - a_c\sin(\theta(t))\,\hat{j}$

Classical (Galilean) Relativity

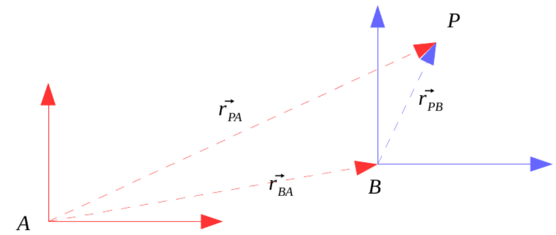
For two inertial* reference frames A and B

Position of a point P: $\vec{r}_{PA}=\vec{r}_{PB}+\vec{r}_{BA}$

Velocity of a point P: $\vec{v}_{PA}=\vec{v}_{PB}+\vec{v}_{BA}$

Acceleration of a point P: $\vec{a}_{PA}=\vec{a}_{BA}$

* Inertial Reference frame : A reference frame in which all laws of physics hold (typically a reference frame that itself is not accelerated)



Statics +Dynamics

	Linear	*Rotational*
Newton's 1st Law	$\vec{F}_{net}=0 \Leftrightarrow \vec{a}=0$	$\vec{\tau}_{net}=0 \Leftrightarrow \vec{\alpha}=0$
Newton's 2nd Law	$\vec{F}_{net}=m\vec{a}$	$\vec{\tau}_{net}=I\vec{\alpha}$
Newton's 3rd Law	$\vec{F}_{AB}=-\vec{F}_{BA}$	$\vec{\tau}_{AB}=-\vec{\tau}_{BA}$

Universal Law of Gravity $F_G=G\frac{m_1 m_2}{r^2}$

Hooke's Law $\vec{F}=-k\vec{x}$

Torque $\tau=rF\sin(\alpha)$, $\vec{\tau}=\vec{r}\times\vec{F}$

Rotational Inertia in General $I=\int x^2\,dm$

Parallel axis theorem $I=I_{com}+mh^2$

Mass - distance L from axis. $I=mL^2$

Beam, hinged at one side $I=\frac{1}{3}mL^2$

Thin rod – perpendicular central axis $I=\frac{1}{12}mL^2$

Solid sphere– central axis $I=\frac{2}{5}mr^2$

Spherical shell – central axis $I=\frac{2}{3}mr^2$

Solid cylinder – central axis $I=\frac{1}{2}mr^2$

Work+Energy

Conservation of Energy	$E_{final}=E_{initial}+W_{by\ NCext}$
Work done by a force	$W=\vec{F}\cdot\vec{s}=F\,s\cos(\Theta)$

NCext: Non Conservative External force

Potential gravitational Energy: PE=mgh

Potential spring Energy PE=1/2kx^2

Linear Kinetic Energy KE= ½mv^2

Rotational Kinetic Energy KE= ½Iω^2

Mechanical Energy ME=PE+KE

Power [Watt] $P=\frac{\Delta W}{\Delta t}=\vec{F}\cdot\vec{v}$

Momentum

	Linear	*Rotational*
Momentum	$\vec{p}=m\vec{v}$	$\vec{L}=\vec{r}\times\vec{p}=m(\vec{r}\times\vec{v})=I\vec{\omega}$
Newton's 2nd Law	$\vec{F}_{net}=\frac{\Delta\vec{p}}{\Delta t}$	$\vec{\tau}_{net}=\frac{\Delta\vec{L}}{\Delta t}$
Conservation of Momentum	$\vec{p}_f=\vec{p}_i+\vec{F}_{ext}\Delta t$	$\vec{L}_f=\vec{L}_i+\vec{\tau}_{ext}\Delta t$

Impulse $\vec{J}=\Delta\vec{p}$

Inelastic collision : *Only momentum is conserved*

Elastic Collision *Momentum and Mechanical energy is conserved*

Waves and Optics

Simple Harmonic Motion

Frequency f [Hz]	F=1/T=ω/2π	Position:	$x(t)=x_m\cos(\omega t+\Phi)$
Angular Frequency ω [rad/s]	ω=2π/T=2πf	Velocity:	$v(t)=\frac{dx}{dt}=-v_m\sin(\omega t+\Phi)$
Period T [s]	T=1/f=2π/ω	Acceleration:	$a(t)=\frac{dv}{dt}=-a_m\cos(\omega t+\Phi)$
Mass and Spring	$\omega=\sqrt{\frac{k}{m}}$, $T=2\pi\sqrt{\frac{m}{k}}$	**Torsion Pendulum**	$T=2\pi\sqrt{\frac{I}{k}}$
restoring force	F= m a(t) = - m ω² x(t)		
Kinetic / Potential Energy	$K=\frac{1}{2}mv(t)^2$, $U=\frac{1}{2}k\,x(t)^2$		
		restoring force	$\tau=-\kappa\Theta$
Simple Pendulum		**Physical Pendulum**	
restoring force	$\tau=-L\,F_g\sin(\Theta)=I\alpha$	restoring force	$\tau=-h_c F_g\sin(\Theta)=I\alpha$
… for small angles	$\omega=\sqrt{\frac{g}{L}}$, $T=2\pi\sqrt{\frac{L}{g}}$	… for small angles	$\omega=\sqrt{\frac{mgh}{I}}$, $T=2\pi\sqrt{\frac{I}{mgh}}$

Mechanical Waves

Wavelength [m]	$\lambda=\frac{2\pi}{k}$	Wave speed [m/s]	$v=\frac{\Delta x}{\Delta t}=\lambda f=\frac{\omega}{k}=\frac{\lambda}{T}$
Angular wave number [1/m]	$k=\frac{2\pi}{\lambda}$	Superposition principle	$y(x,t)=y_1(x,t)+y_2(x,t)$
Soft reflection (free end, denser to less dense)	No phase shift	Hard reflection (fixed end, less dens to denser medium	π
Transverse waves	$f(x,t)=\boldsymbol{h}(kx\pm\omega t)$ $f(x,t)=y_m\sin(kx\pm\omega t+\Phi)$ +: moving left - : moving right	**Longitudinal waves (Sound)**	$s(x,t)=s_m\cos(kx\pm\omega t+\Phi)$ $\Delta p(x,t)=\Delta p_m\sin(kx\pm\omega t+\Phi)$
		Sound Level (Decibel) $I_0=10^{-12}W/m^2$	$\beta=10\,dB\cdot\log_{10}(\frac{I}{I_0})$, $+1\,dB=x\,10$
Particle velocity / acceleration:	$u=\frac{\delta y(x,t)}{\delta t}$, $a=\frac{\delta u(x,t)}{\delta t}$	Intensity I	$I=\frac{P}{A}$ $I=10^{\frac{\beta}{10}}\cdot I_0$
Wave speed:	$v=\sqrt{\frac{\tau}{\mu}}$ τ=Tension, μ= linear density	Wave speed:	$v=\sqrt{\frac{B}{\rho}}$, B: Bulk modulus $B=\frac{-\Delta p}{\Delta V/V}$

...Interference identical waves, shifted by Φ(=Δx/λ), traveling in the same direction

$y'(x,t)=[2y_m\cos(\frac{\Phi}{2})]\sin(kx-\omega t+\frac{\Phi}{2})$

$s'(x,t)=[2s_m\cos(\frac{\Phi}{2})]\cos(kx\pm\omega t+\frac{\Phi}{2})$

(for different amplitudes, use the Phasor-Diagram)

Constructive:	ϕ = 2πm or Δx=mλ, m=0, 1, 2, ...	Deconstructive:	ϕ = πm or Δx=mλ/2, m= 1, 3, 5,...
		Beats	$f_{beat}=f_1-f_2$
		Doppler Effect (classic)	$f'=f\,\frac{v\pm v_d}{v\mp v_s}$

...Standing waves (identical waves traveling in opposite directions)

$y'(x,t)=[2y_m\sin(kx)]\cos(\omega t)$

nodes: no amplitude
antinodes: max amplitude

String, both ends fix	$\lambda_n=\frac{2L}{n}, n=1,2,3,4...$	Pipe, both ends open	$\lambda_n=\frac{2L}{n}, n=1,2,3,4...$
		Pipe, open-closed	$\lambda_n=\frac{4L}{n}, n=1,3,5...$

Geometrical Optics

Snells Law (Law of refraction)	$n_1 \sin(\Theta_1) = n_2 \sin(\Theta_2)$, $\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$	Thin lens equation: Diopters D = 1/f	$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, $m = \frac{h_I}{h_O} = \frac{-q}{p}$ p: Object distance q: Image distance (<0: Virtual, >0: Real) f: focal distance (f<0: Diverging, f>0: Converging)
Law of reflection	$\Theta_i = \Theta_r$	Ray diagrams for Lenses:	

Ray diagrams for Lenses:

1. A ray through the center goes straight through.
2. A ray parallel to the axis goes through the focal point.
3. A ray through the focal point becomes parallel to the axis.

Ray diagrams for curved mirrors:

1. Any ray reflects according to the law of reflection at the tangent to the curve
2. A ray parallel to the axis goes through the focal point.
3. A ray through the focal point becomes parallel to the axis.

f=radius/2

Wave Optics

Phase difference due to path traveled in different medium (number of wavelengths)	$N_2 - N_1 = \frac{L}{\lambda_0}(n_2 - n_1)$	Thin film interference	Add phase shift due to reflection and path difference to determine constructive and destructive interference.
Double Slit Interference		Double Slit Diffraction	
Maxima:	$d \cdot \sin(\Theta) = m\lambda$, $m = 0, 1, 2...$	*Maxima*	Complicated
Minima	$d \cdot \sin(\Theta) = (m + 0.5)\lambda$, $m = 0, 1, 2...$	*Minima*	$a \cdot \sin(\Theta) = m\lambda$, $m = 1, 2, 3...$
Diffraction by a circular aperture	$\sin(\Theta) = 1.22 \frac{\lambda}{d}$	Diffraction Grating	Grating spacing: d=width/(# of gratings N)
Rayleigh's Criterion for resolvability	$\Theta_R = \arcsin\left(1.22 \frac{\lambda}{d}\right)$	*Maxima Lines*	$d \cdot \sin(\Theta) = m\lambda$, $m = 0, 1, 2...$
		… First minimum	$N\, d \sin(\Delta\Theta) = \lambda$
		… Half line width	$\Theta_{hw} = \frac{\lambda}{N\, d \cos(\Theta)}$
		Resolving power	$R = \lambda_{avg} / \Delta\lambda = Nm$

Modern Physics

Nuclear Physics

$^{A}_{Z}X$ A: Mass number (Protons+Neutrons)

Z: Atomic number (charge of the nucleus)
X: Chemical Symbol

Radius of the nucleus $r = r_0 A^{1/3}$ r_0: 1.2 fm (1.2 x 10^{-15} m)

Nuclear Equation: Mass number and Charge is conserved (Mass is not)

Binding Energy: $E_{BE} = \Delta m \cdot c^2 = [(Z \cdot (m_p + m_e) + N \cdot m_n) - m_{tot}] \cdot c^2$

Z:	Number of protons
N:	Number of neutrons
m_{tot}:	Total mass of the isotope
$m_p + m_e = m_{H-1}$:	Mass of a proton + an electron (1.007 825 u)
m_n:	Mass of a neutron (1.008 665 u)
c^2:	931.494013 MeV/u

Radiation / Decay

	Event in Nucleus	*Radiated particles*
α-Decay	$-^{4}_{2}He$	$^{4}_{2}He$
β-Decay	$^{1}_{0}n \rightarrow ^{1}_{+1}p + ^{0}_{-1}e + \bar{\nu}_e$	$^{0}_{-1}e + \bar{\nu}_e$
Positron-Decay	$^{1}_{+1}p \rightarrow ^{1}_{0}n + ^{0}_{+1}e + \nu_e$	$^{0}_{+1}e + \nu_e$
γ-Decay	*Becomes stable*	$^{0}_{0}\gamma$

Number of radioactive nuclei: $N(t) = N_0 e^{-\lambda t}$

Decay rate [Bq]: $R(t) = \lambda N(t) = R_0 e^{-\lambda t}$

Units of radiation

Activity	Decay events per second	→	Becquerel [Bq] = disintegrations/ s	1 Curie = 3.7 x 10^{10} Becquerel
Absorbed Dose	Absorbed energy	→	Gray [Gy] = Joules / kg	1 Gray = 100 rad
Biological Damage	Effect on humans	→	Sievert [Sv] = Gray x Factor	1 Sievert = 100 rem

Quantum Physics

Energy of a photon $E = hf$

Momentum of a photon $p = \frac{hf}{c} = \frac{h}{\lambda}$

De Broglie Wavelength $\lambda = \frac{h}{p}$

Photoelectric Effect: $K_{max} = eV_{stop}$, $hf = K_{max} + \Phi$, $V_{stop} = (\frac{h}{e})f - \frac{\Phi}{e}$

Heisenbergs Uncertainty principle: $\Delta x \cdot \Delta p_x \geqslant \hbar$
$\Delta y \cdot \Delta p_y \geqslant \hbar$
$\Delta z \cdot \Delta p_z \geqslant \hbar$

$\hbar = h/(2\pi)$

Special Relativity

First postulate: The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other. Second postulate: The speed of light in vacuum has the same value c in all directions and in all inertial reference frames.		**Simultaneity is relative,** it depends on the motion of the observer	Two observers in relative motion will in general not agree whether two events are simultaneous or not.
		Lorentz Factor:	$\gamma=\frac{1}{\sqrt{1-(v/c)^2}}$
Proper time t_0	Time interval between two events occurring at the same location in an inertial reference frame.	Time dilation:	$\Delta t=\gamma\,\Delta t_0$
Proper length L_0:	The distance between two points measured in an inertial reference frame at rest relative to both points.	Length contraction:	$L=\frac{L_0}{\gamma}$
Rest Mass	The mass of an object as measured in a reference frame not moving relative to the object (the only mass that is directly measurable)	Momentum:	$p=m v\cdot\gamma$
Lorentz Transformation A system x'y'z't' moving at a speed in x-direction relative to the system xyzt.	$x'=\gamma(x-vt)$ $y'=y$ $z'=z$ $t'=\gamma(t-\frac{v}{c^2}x)$	Kinetic Energy Rest Energy	$KE=\gamma m c^2-mc^2$ $E_0=m c^2$
Velocities	$u'=\frac{u-v}{1-\frac{uv}{c^2}}$	Minkowski Diagram	Graphical visualization of time dilation and length contraction
Doppler Effect	$f=f_0\sqrt{\frac{c-v}{c+v}}$ $f=f_0\,\gamma(1-\frac{v}{c})$	General Relativity	Generalization of the special relativity (allowing accelerations). Description of Gravity as a property of space-time

Electricity

Charge, Coulombs Force, Electric Field and Potential

Charge is quantized: Q=N*e [C]

Current: i=dQ/dt [A]

Coulombs Law:

$$\vec{F}=k\frac{q_1 q_2}{|(r)|^2}\cdot\frac{\vec{r}}{|(r)|} \quad \text{[N]}$$

$$\vec{F}_E=-\frac{\partial U}{\partial \vec{s}}$$

Electric Field:

$$\vec{E}=\frac{\vec{F}_e}{q_t}=k\frac{q_s}{r^2}\cdot\hat{r} \quad \text{[N/C] = [V/m]}$$

$$\vec{E}_{tot}=\vec{E}_1+\vec{E}_2+\ldots\vec{E}_n$$

$$\vec{E}=-\frac{\partial V}{\partial \vec{s}}$$

Potential Energy of a conservative force: $\Delta U=U_{final}-U_{initial}=-W_{done\,by\,force}$ [J]

(Electric) Potential Difference: $\Delta V=\frac{\Delta U_{el}}{q}=-\int \vec{E}\cdot d\vec{s}$ [V]

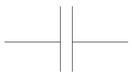
Electric Potential Energy: $\Delta U_{el}=q\Delta V=-W_{el}=-\int \vec{F}_{el}\vec{s}=-\int q\vec{E}\,d\vec{s}$ [J]

Electric Potential due to a point charge: $V=k\frac{q}{r}$

Electron Volt: $1eV=1.6022\cdot10^{-19}J$

Capacitors

Capacitance: $C=\frac{Q}{V}$ [F] = Farad (Coulombs/Volt)



Capacitance in Series:

$$\frac{1}{C_{eq}}=\sum\frac{1}{C_i}$$

$$Q=Q_1=\ldots=Q_n \quad , \quad V=V_1+\ldots+V_n$$

Parallel plate capacitor:

Field: $E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{\varepsilon_0 A}$

Potential: $|V|=E\cdot d$

Capacitance: $C=\frac{\varepsilon_0 A}{d}$

Capacitance in Parallel:

$$C_{eq}=\sum C_i$$

$$Q=Q_1+\ldots+Q_n \quad , \quad V=V_1=\ldots=V_n$$

Energy stored in a capacitor: $U=\frac{Q^2}{2C}=\frac{1}{2}QV=\frac{1}{2}CV^2$

Energy density in the field of a capacitor: $u=\frac{1}{2}\varepsilon_0 E^2[J/m^3]$

Current and Resistance

Current: $i=\frac{dq}{dt}$ [A]

Technical direction: + to -
Real direction of electrons in metals: - to +

Current density: $J=\frac{i}{A}=n\cdot e\cdot v_d$

A: Cross section area
n: Charges per Volume / free electron density
e: Charge of an electron
v_d: Drift Speed: Speed of the charges in a wire

Resistance: $R=\frac{V}{I}$ Ohm [Ω]=[V/A]

Resistivity:

$$\rho=\frac{E}{J} \quad [\Omega\text{m}]$$

$$R=\frac{\rho\cdot length}{area}$$

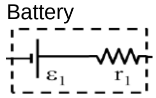
Power dissipated in a resistor: $P=IV=I^2\cdot R=P=\frac{V^2}{R}$ Watt [W]=[J/s]

Conductivity: $\sigma=\frac{1}{\rho}$

Electromotive Force (EMF)

Ideal EMF: Can deliver as much current as required to maintain a fixed potential difference.

Battery: Has an internal resistance.
As soon as current flows, the terminal Voltage is different from the EMF.
Some Energy is lost (transformed to heat) in the internal resistance.

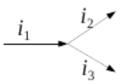


Power delivered of an ideal EMF: $P=EMF\cdot I$

Kirchoff's Rules

Junction Rule

$$i_1+i_2+i_3+...=0$$



The sum of all currents at a junction is zero.

Sign rules:
Current entering the junction: +
Current leaving the junction: -

Loop Rule

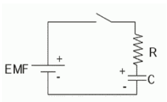
$$\Delta V_1+\Delta V_2+...+\Delta V_n=0$$

The sum of all voltages around a closed loop is zero.

Sign rules:
Going through a resistor in the same direction as the current : - ΔV
Going through a resistor against the current : + ΔV
Going through an EMF from – to + : + ΔV
Going through an EMF from + to - : - ΔV

RC-Circuits

Charging



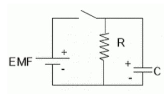
$$q(t)=Q_f(1-e^{\frac{-t}{\tau}})$$

$$V(t)=\frac{q}{C}=V_f(1-e^{\frac{-t}{\tau}})$$

$$i(t)=\frac{dq}{dt}=I_0 e^{\frac{-t}{\tau}}$$

with
Final Charge: $Q_f=CV_f$
Initial Current: $I_0=\frac{V_f}{R}$
Time constant: $\tau=RC$
Final voltage: Depends on the circuit.

Discharging



$$q(t)=Q_i e^{\frac{-t}{\tau}}$$

$$V(t)=\frac{q}{C}=V_i e^{\frac{-t}{\tau}}$$

$$i(t)=\frac{dq}{dt}=-(\frac{q_i}{\tau})e^{\frac{-t}{\tau}}=I_i e^{\frac{-t}{\tau}}$$

with
Initial Charge: $Q_i=CV_i$
Initial Current: $I_i=\frac{V_i}{R}$
Time constant: $\tau=RC$

Magnetism

Magnetic Fields and effect on moving charges

Tesla [T]	SI-Unit of the magnetic field $T=\frac{N\cdot s}{C\cdot m}$	Gauss [G]	Old unit of the magnentic field (not SI). 1 Gauss = Field strength of the magnetic field of the earth at the surface $1T=10^4 G$
Lorentz Force	$\vec{F}_B=q\cdot\vec{v}\times\vec{B}$ $F_B=\lvert q\rvert\cdot v\cdot B\cdot\sin(\Phi)$	Charged particles circulating in a magnetic field	$\lvert q\rvert\cdot v\cdot B=m\frac{v^2}{R}$
Fore on a current carrying wire	$\vec{F}_B=i(\vec{L}\times\vec{B})$ $F_B=i\,L B\sin(\Phi)$	Cyclotron	$\lvert q\rvert B=2\pi\, m f_{osc}$
Magnetic Dipole Moment	Unit: [A m²] or [J/T] Magnetic Dipole Moment of a coil: $\mu=N\,i\,A$ N: Number of turns (Loops) i: Current A: Area of the loop	Velocity selector $\vec{E}\perp\vec{B}$	$\lvert q\rvert E=\lvert q\rvert v\,B\sin(90°)\rightarrow v=E/B$
Torque on a Magnetic Dipole	$\vec{\tau}=\vec{\mu}\times\vec{B}$		
Potential Energy of a Magnetic Dipole	$U(\Theta)=-\vec{\mu}\cdot\vec{B}$ $U(\Theta)=-\mu\cdot B\cos(\Theta)$		

Magnetic Fields due to currents

Law of Biot-Savart	*Vector Form:* $d\vec{B}=\frac{\mu_0}{4\pi}\cdot\frac{i\,d\vec{s}\times\hat{r}}{r^2}$. $\hat{r}=\frac{\vec{r}}{\lvert r\rvert}$	*Magnitude Form:*	$dB=\frac{\mu_0}{4\pi}\cdot\frac{i\cdot ds\cdot\sin(\theta)}{r^2}$
Magnetic field due to long straight wire at distance r	$B=\frac{\mu_0 i}{2\pi r}$	Magnetic field due to straight wire of finite length	$B=\frac{\mu_0 i}{4\pi r}\cdot\sin(\alpha)$ Angle: Section of the wire seen from the point. Wire: Must start perpendicular to the point
Magnetic field due to circular arc of wire	$B=\frac{\mu_0 i\Phi}{4\pi r}$ Angle in **rad!**	Force between two parallel currents	$F_{BA}=F_{AB}=i_B L B_A\sin(90°)=\frac{\mu_0 L i_A i_B}{2\pi r}$
Magnetic Field of a solenoid	If L>> D $B=\frac{\mu_0 N i}{L}=\mu_0 n\,i$ otherwise $B=\frac{\mu_0 N i}{\sqrt{(L^2+D^2)}}$ N= turns L= length n= turns per meter D= Diameter		

Currents due to magnetic fields

Magnetic Flux	$\Phi=B\cdot A\cdot\cos(\Theta)$ Unit: [Tm²] = [Wb]	Faraday's Law	$\lvert EMF_{Induced}\rvert=\left\lvert N\frac{d\Phi}{dt}\right\rvert$ N= turns
Transformer	$V_s=\frac{N_s}{N_p}V_p$, $I_s=\frac{N_p}{N_s}I_p$	Lenz's Law	The induced EMF creates a current I and a magnetic field B that oppose the change in magnetic flux

Inductors

Inductance L	[VsA^{-1}] = [H] (Henry)		
Self-Inductance of an inductor	$EMF=-L\frac{dI}{dt}$	Energy stored in an inductor.	$E=\frac{1}{2}LI^2$
RL-Circuit “turning on”	$I(t)=I_0(1-e^{\frac{-t}{\tau}})$, $\tau=\frac{L}{R}$	RL-Circuit “turning off”	$I(t)=I_0 e^{\frac{-t}{\tau}}$, $\tau=\frac{L}{R}$

AC Current

Alternating current	$V(t)=V_0\sin(\omega t)$ $I(t)=I_0\sin(\omega t-\varphi)$ ω : Angular Angular Frequency [rad/s] $\omega=2\pi f$ φ ; Phase shift (Delay of current behind voltage)	Root mean square values	$V_{rms}=\frac{V_0}{\sqrt{(2)}}$ $I_{rms}=\frac{I_0}{\sqrt{(2)}}$
Average Power	$P_{avg}=V_{rms}I_{rms}\cos(\varphi)$	Instantaneous Power	$P(t)=V(t)I(t)$
Impedance	$Z=\frac{V_0}{I_O}=\frac{V_{rms}}{I_{rms}}$ Ohm [Ω]=[V/A]	RCL Circuit - Series	$Z=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$ $\tan(\varphi)=\frac{(\omega L-\frac{1}{\omega C})}{R}$ *Resonance frequency:* $\omega=\frac{1}{\sqrt{LC}}$
Inductor (Reactance X_L)	$Z=\omega L$, $\varphi=\frac{\pi}{2}$ (V leads, I behind)	RCL Circuit – Parallel	$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+(\omega C-\frac{1}{\omega L})^2}$ $\tan(\varphi)=R(\frac{1}{\omega L}-\omega C)$
Capacitor (Reactance X_C)	$Z=\frac{1}{\omega C}$, $\varphi=-\frac{\pi}{2}$ (I leads, V behind)		
Resistor (Impedance)	$Z=R$, $\varphi=0$		

Physical Science

Significant Figures + Measurement

Measurements	The last digit of a measurement is always estimated (meaning, it is not certain)	*Example :*	*1.05 cm → meaning, it could have been 1.04 or 1.06 cm*
Significant Figures	A number is significant when it is: - not a zero - a zero between non-zero digits - a zero after a-non zero on the right of the decimal point or on the left of the decimal point - in the coefficient of a scientific number	*Examples :*	*123 → 3 significant figures* *1001 → 4 significant figures* *1.00 → 3 significant figures* *100. → 3 significant figures* *1.00 x 10^3 → 3 significant figures*
	A zero is not significant when it is: - on the left of all non-zero digits - on the right of all non-zeros in a number without decimal point	*Examples:*	*0.03 → 1 significant figure* *100 → 1 significant figure*
Exact numbers	Counted or definitions	Rounding Rules	>= 0.5 → round up, < 0.5 → round down

Calculation rules for significant figures

Addition / Subtraction	Give the result with the fewest <u>decimals</u>	*Examples:*	*1.00 + 2.3 = 3.3* *5.5 - 0.50 = 5.0*
Multiplication / Division	Give the result with the fewest <u>significant figures.</u>	*Examples:*	*2.00 x 2.0 = 4.0* *4 / 2.00 = 2*

Density + Matter

Density	Density = mass / Volume	**States**	
Matter	Has space and occupies volume	Solid	Definite shape and volume
Element	A collection of atoms of one type.	Liquid	Takes shape of the container, definite volume
Compound	A combination of atoms of different types	Gas	Takes shape and volume of the container
Mixture	A mix of different element or compounds	*Heterogeneous*	*Differences in the mixture are visible*
		Homogenous	*No difference in the mixture is visible. It appears identical throughout.*
Chemical Property	Ability to interact with other substances		
Chemical Change	New substances with new properties are formed	*Signs:*	*Color change, bubbles, heat produced or adsorbed*
Physical Property	Shape, State, ...		
Physical Change	Change in shape and state, no new substances are formed		

Atoms

Dalton	Atoms are the smallest unit of matter. All atoms of one element are identical.		*Billiard Ball*
Thomson	Atoms are made of subatomic elements that are uniformly distributed		*Plum pudding*
Rutherford	Atoms are made of a small, dense, positive nucleus (protons and neutrons), and negative electrons orbiting the nucleus		*Planetary model*
Rutherford-Bohr	Like Rutherford, but the electrons are only found in distinct orbits.		*Planetary model with only few existing electron orbits corresponding to energy levels.*
Isotopes	Atoms of the same element but with a different mass number (= different number of neutrons)	Mass number	Number of Protons + Neutrons

Periodic Table (see Annex 1)

Periodic Table	Atoms are • ordered by ascending proton number (= atomic number) • in groups (top down) of similar properties • periods (horizontal)	Atomic Number Symbol Atomic mass Element Name	**Atomic Number :** Number of protons **Symbol:** One or two letters. First letter is a capital letter **Atomic mass :** Average mass of all isotopes of the element (weighted by natural occurance) The unit of the number is amu/atom AND g/mol
Alkali Metals (G1 excluding H)	Highly reactive, Soft with low melting point.	Metals	*Left side of the Periodic Table*
Alkaline Earth Metals (G2)		Non-Metals	*Right side of the Periodic Table*
Transition Elements (G3-12)		Metalloids	*Zig-Zag line between Metals and Non-Metals. Semiconductors*
Halogens (G7)	Highly reactive gases.	**Diatomic Molecules**	Elements that in their elemental form come in pairs of two: H_2, N_2, 0_2, F_2, Cl_2, Br_2, I_2 (Have N O Fear of Ice Cold Bear)
Noble gases (G8)	Do not react with any other elements.		

Chemical Bonding

Valence electrons	Electrons in the highest energy level of an atom (=Group Number for elements other than transition elements)		Lewis – Diagram	Shows the valence electrons of an element
Octet Rule	All elements want to achieve the valence electron configuration of the closest noble gas.		Metals	Achieve noble gas configuration by giving up valence electrons
			Non-Metals	Achieve noble gas configuration by increasing the number of valence electrons
			Metalloids	Can achieve noble gas configuration by increasing or decreasing the number of valence electrons
Ionic Bond	Transfer of electrons	→	Metal + Nonmetal, Polyatomic Ion + Nonmetal, Metal + Polyatomic Ion, Polyatomic Ion + Polyatomic Ion	
Covalent Bond	Sharing of electrons	→	Non-metal + Non-metals	
Metallic Bond	Transferring of electrons in an electron cloud	→	Metals (making them conducting)	

Nomenclature

Ionic Compounds	Metal can only have one charge	→	Metal Name + Non-Metal Name + ide
	Metal can have different charges	→	Metal Name (Roman Numeral for charge) + Non-Metal name + ide
	Anion is a polyatomic Ion	→	As above, but without the -ide
Polyatomic Ions	Less oxygen	→	Name of first element + ite
	More oxygen	→	Name of first element + ate
	With hydrogen	→	Bi + Name
Covalent Compounds	Greek prefixes for numbers	→	Greek Prefix + Non-Metal + Greek Prefix + Non-Metal + …. + ide
Acids	See acid section		

Moles and Chemical Reactions

Mol	1 mol = 6.022×10^{23} particles	Definition:	1 mol is the number of particles in 12.01 g of carbon.
Molar mass	Mass (in g) of one mol of a particles of a substance	For elements:	Mass given in periodic table (attention to diatomic substances!)
		For compounds:	Example $C0_2$: Molarmass = 1x12.01gg + 2x16.00 g
Chemical Reactions	Reactants → Products	- No atoms are gained or lost - No mass is gainer or lost	
Combustion reaction	Fuel + Oxygen → Water and CO_2		

Solutions

Solution	A homogenous mixture of two or more substances	Solute	Substance of a solution that is in the smaller amount
		Solvent	Substance of a solution that is in the larger amount
		Solubility	The maximum amount of solute that a solvent can dissolve
Molarity	$M=\frac{n}{V}=\frac{mol\ of\ solute}{L\ of\ solution}$	Dilution	$M_1V_1=M_2V_2$
Concentration (by mass)	$C(\%m/m)=\frac{g\ of\ solute}{g\ of\ solution}$		

Acids and Bases

Acid	Generates H^+ ions in water (producing H_30^+)	Binary Acids (H^+ + NM^-)	Hydro + NM + ic + acid
		Oxy-Acids ((H^+ + Ion^-)	Ion ends in -ate → ….. -ic acid (No hydro!) Ion ends in -ite → ….. -ous acid (No hydro!)
Reactions	Acid + Metal → H_2 + Salt		
	Acid + Carbonate → CO_2 + Water + Salt		
Base	Generates OH^- ions in water / Accepts H^+		
Reactions	Acid + Base → Water + Salt		
pH	$pH=-\log(\frac{mol\ H_3O}{L})$, $\frac{mol\ H_3O}{L}=1x10^{-pH}$	pH 0....7	Acidic
	# of decimal places of pH = # of sig. fig. of the coefficient	pH 7....14	Basic
pOH	$pOH=-\log(\frac{mol\ OH}{L})$, $\frac{mol\ OH}{L}=1x10^{-pOH}$	pH + pOH= 14	

Radioactivity and Nuclear Physics

→ See Modern Physics

Transformation of Energy

Work	$W = F_{II} \cdot s = F s \cos(\Theta)$	Potential gravitational Energy:	PE=mgh
Weight	$F_g = mg$	Linear Kinetic Energy	KE= ½mv²
Conservation of Energy	The total amount of energy of a closed system is always constant.	Mechanical Energy	ME=PE+KE
		Thermal Energy	Energy of random motion of atoms and molecules
Endothermic	Adsorbing energy	Heat (transfer of Thermal Energy)	$Q = \Delta E_{th} = mc \Delta T$ c: Specific heat capacity
Exothermic	Releasing energy		

Electricity and Magnetism

→ **See sections on Electricity and Magnetism**

Genetics

Genetics	The science of genes and heredity	Genotype	Genetic makeup
DNA	Deoxyribonucleic acid, blueprint of organisms	Phenotype	Visible traits (how the genotype is expressed)
Chromosomes	Organized structure of DNA	Mitosis	Normal cell division creating exact copies (full chromosome set)
Gene	Chunks of DNA containing the information for a character trait	Meiosis	Cell division for reproduction, half set of chromosomes
Allele	Variation of a gene.		
Dominant Allele	Hides the effect of another allele of the same gene	Homozygous	Having two identical alleles of a gene
Recessive Allele	Effect hidden when a dominant allele of the same gene is present	Heterozygous	Having two different alleles of a gene
Punnet Square	Diagram to predict the outcome of cross breeding	Law of segregation	Each parent only gives one allele to the offspring

Constants and other properties

Constants

Mechanics		Electricity and Magnetism		Modern Physics and Waves	
Universal Gravity constant G	6.67 10^{-11} Nm^2/kg^2	Unit charge e	1.602×10^{-19} C	Speed of light in vacuum c	299792458 m/s
g	9.8 m/s	k	$9.00\times10^9 N \bullet m^2/C^2$	Planck Constant h	6.63×10^{-34} Js
		μ_0	$4\pi\times10^{-7}$H/m	"h-bar"	$\hbar = h/(2\pi)$
		ε_0	8.85×10^{-12}F/m		

Mass

Proton	1.007276 u	H-1	1.007 825 u	He-4	4.002 603 u	Zr-97	96.9109531 u	U-238	238.0507882
Neutron	1.008 665 u	H-2				Te-109	108.92742 u	U-235	235.043929 u
Electron	0.0005485799 u	H-3	3.016 05 u			Xe-113	112.93334 u	Th-234	234.0436012
C-12	12.0 u					Cs-133	132.905429 u	Ce-140	139.905434 u
						Xe-133	132.9059107 u	Te-137	136.92532 u
								Nd-144	143.910083 u

1 u = 1.6605×10^{-27} kg

Half-life

U-238	4.5×10^9 y	C-14	5730 y	H-3	12.3 y	Ir-192	74 d	Tc-99m	6 h
K-40	1.3×10^9 y	Ra-226	1600 y			Fe-59	44 d		
Ra-226	1.6×10^3 y					Cr-51	28 d		
						I-131	8 d		

Units

1 light year = 9.46×10^{15} m	1 u = 1.6605×10^{-27} kg	1 eV = 1 .602 x 10^{-19} J	1 mol = 6.022 x 10^{23} particles
Kelvin	Celsius + 273	$1u\cdot c^2$ =931.494MeV	1 cal = 4.184 J
Celsius	5/9 (Fahrenheit-32)	1 T = 10^4 G	1 Cal = 1 kcal = 1000 cal

Prefixes

peta	P	10^{15}	deci	d	10^{-1}
tera	T	10^{12}	centi	c	10^{-2}
giga	G	10^9	milli	m	10^{-3}
mega	M	10^6	micro	μ	10^{-6}
kilo	k	10^3	nano	n	10^{-9}
			pico	p	10^{-12}
			femto	f	10^{-15}

Numbers (Greek, Latin)

1	Mono	I
2	Di	II
3	Tri	III
4	Tetra	IV
5	Penta	V
6	Hexa	VI
7	Hepta	VII
8	Octa	VIII
9	Nona	IX
10	Deca	X

Annex 1 - Periodic Table