Fluid Dynamics

- **Flow rate**
- **Equation of continuity**
- **Bernoulli’s principle**
- **Venturi effect**
- **Some applications**

Flow rate

$$Q=\frac{V}{t}$$

Q	Flow rate	**m³/s**	or L/s	or L/min
V	Volume	**m³**	or L	
t	Time (difference)	**s**		or min

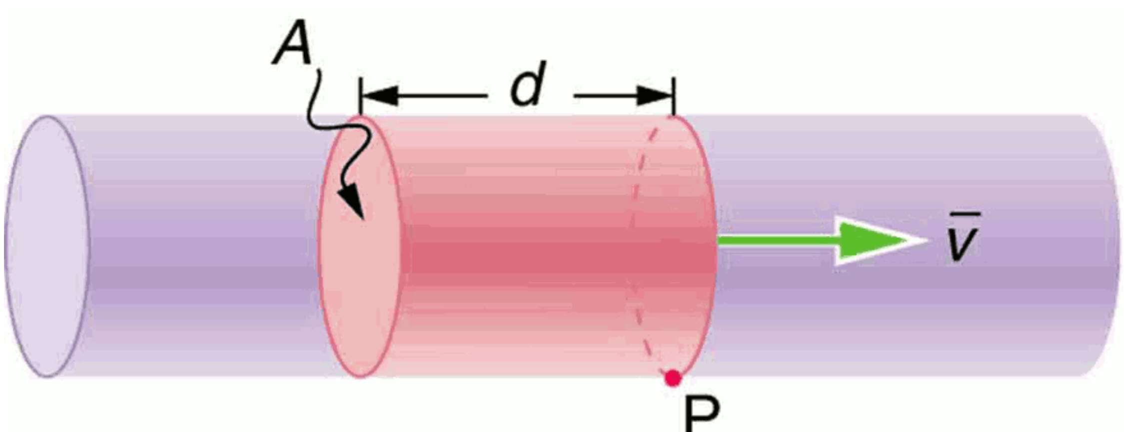


Image: OpenStax, College Physics. OpenStax CNX. 4. Nov. 2016
http://cnx.org/contents/Ax2o07Ul@9.39:g9Jw9bqg@4/Flow-Rate-and-Its-Relation-to-

Volume=Cross section Area times distance

Speed = distance over time

$$V=A\cdot d$$

$$v=\frac{d}{t}$$

$$Q=\frac{A\cdot d}{t}=A\cdot v$$

Equation of continuity for incompressible fluids

$$Q_1 = Q_2$$
$$\rightarrow v_1 A_1 = v_2 A_2$$

Q	Flow rate
v	speed
A	Cross section area

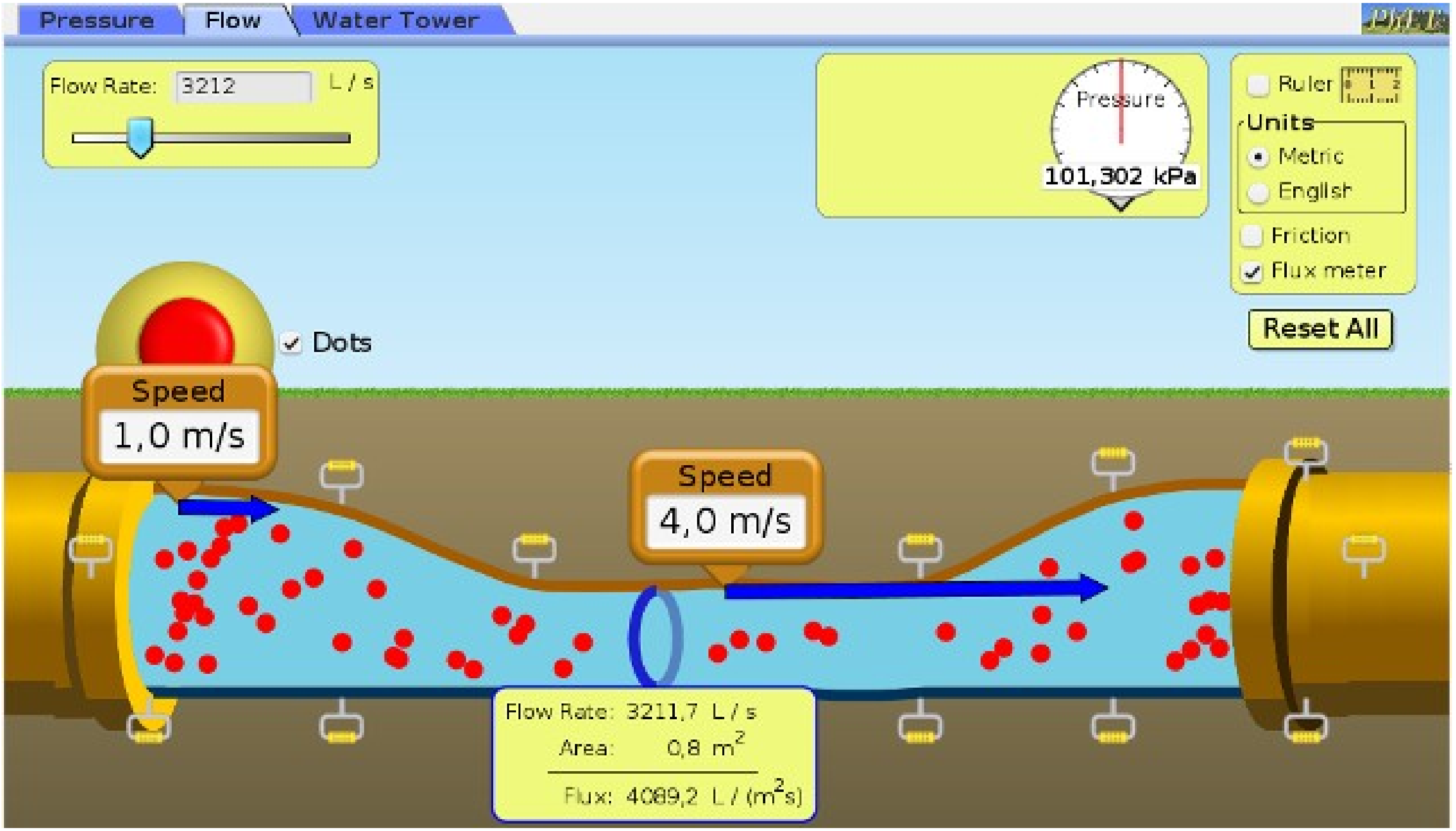


Image: Phet Simulation „Fluid and Pressure Flow“ https://phet.colorado.edu/en/simulation/legacy/fluid-pressure-and-flow

Bernoulli's principle – Conservation of mechanical energy for fluids

Conservation of mechanical energy:

$$PE_2 + KE_2 = PE_1 + KE_1 + W_{ext}$$

$$mgh_2 + \frac{1}{2} m v_2^2 = mgh_1 + \frac{1}{2} m v_1^2 + F_{ext} d$$

using $m = \rho V$ and $F = (P_1 - P_2) \cdot A$ leads to:

$$\rho V g h_2 + \frac{1}{2} \rho V v_2^2 = \rho V g h_1 + \frac{1}{2} \rho V v_1^2 + (P_1 - P_2) A d$$

Then dividing by V (note V=Ad) gives:

$$\rho g h_2 + \frac{1}{2} \rho v_2^2 + P_2 = \rho g h_1 + \frac{1}{2} \rho v_1^2 + P_1$$

Bernoulli's Principle

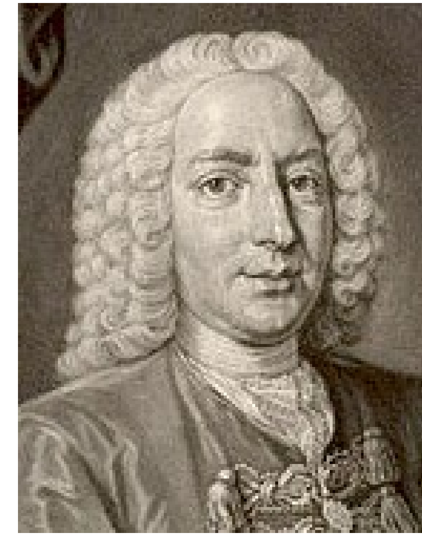


Image:
Daniel Bernouli (1700-1782)
painted by J. J. Haid
[Public Domain]

Venturi effect

Bernoulli's Law

$$\rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2 = \rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1$$

For constant height:

$$\frac{1}{2}\rho v_2^2 + P_2 = \frac{1}{2}\rho v_1^2 + P_1$$

Venturi effect

Image:
Giovanni Battista Venturi
(1746-1822)
[Public Domain]

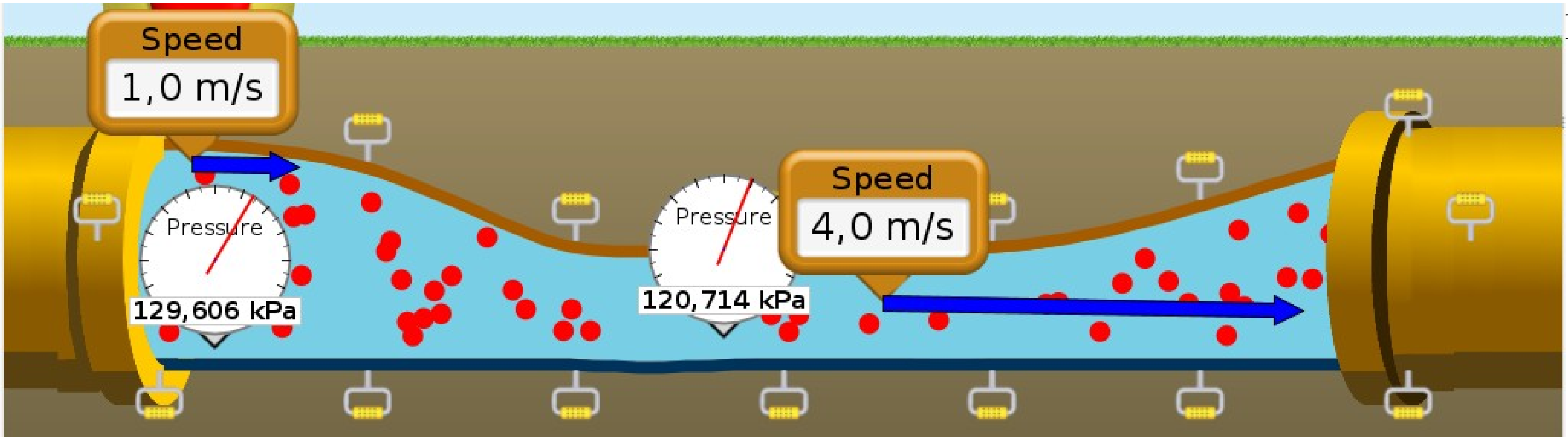


Image: Phet Simulation „Fluid and Pressure Flow“
https://phet.colorado.edu/en/simulation/legacy/fluid-pressure-and-flow

Water Tower

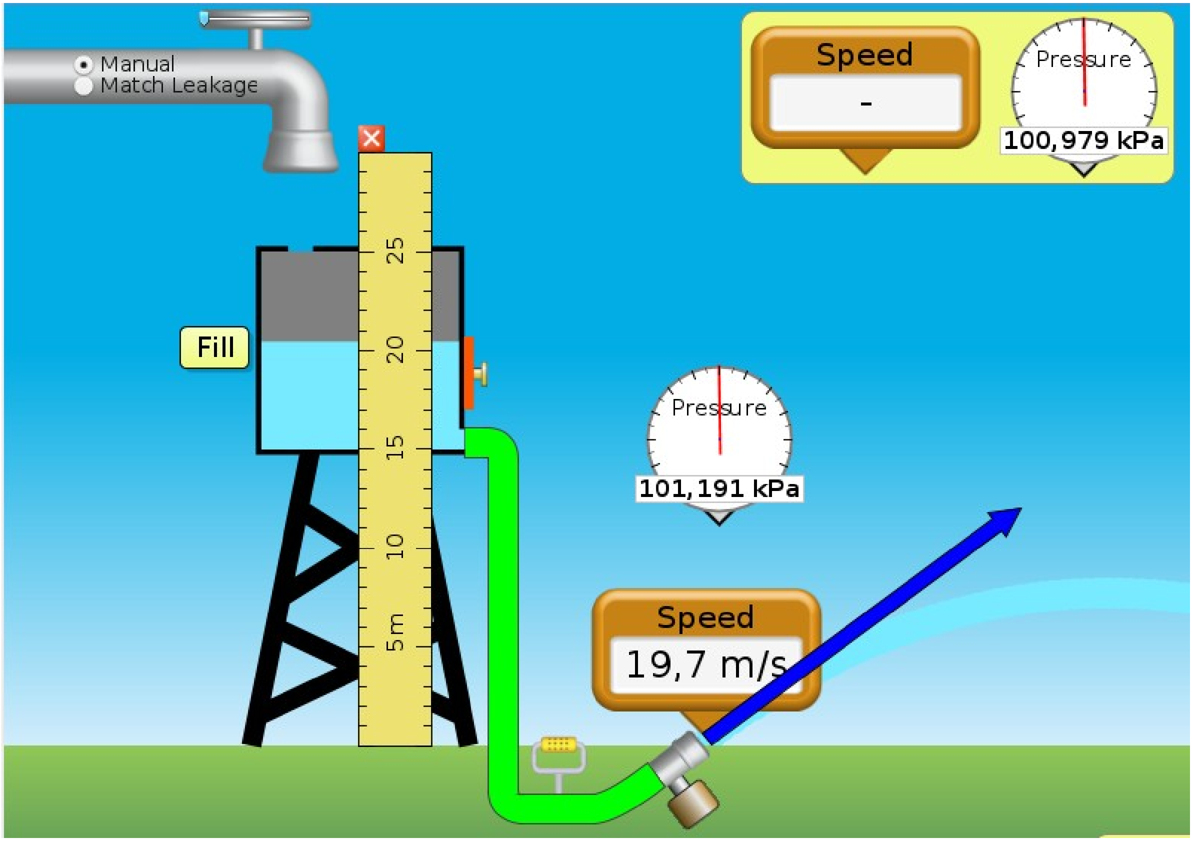


Image: Phet Simulation „Fluid and Pressure Flow“
https://phet.colorado.edu/en/simulation/legacy/fluid-pressure-and-flow

The higher the water level,
the higher the speed at the hose

Bernoulli's equation:

$$\rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2 = \rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1$$

The pressures in the tower and outside the hose are equal:

$$\rho g h_2 + \frac{1}{2}\rho v_2^2 = \rho g h_1 + \frac{1}{2}\rho v_1^2$$

The speed of the water in the tank is 0:

$$\rho g h_2 + \frac{1}{2}\rho v_2^2 = \rho g h_1$$

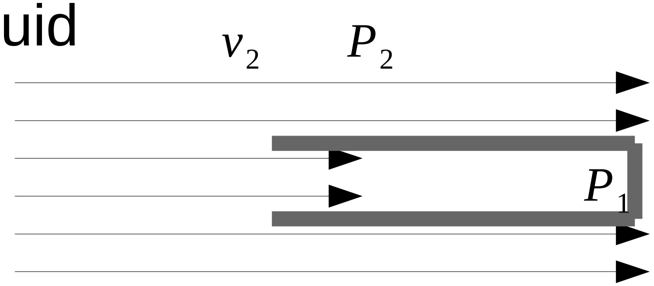
Solve for v:

$$v_2 = \sqrt{2 g (h_1 - h_2)}$$

(Compare with conservation of energy)

Some Applications

Using Pitot tubes to measure the speed of the fluid



The fluid enters a tube that is closed on one end.

The fluid comes to a complete stop at the closed end.

From the pressure difference between the closed end of the tube and the outside, the fluid speed can be determined.

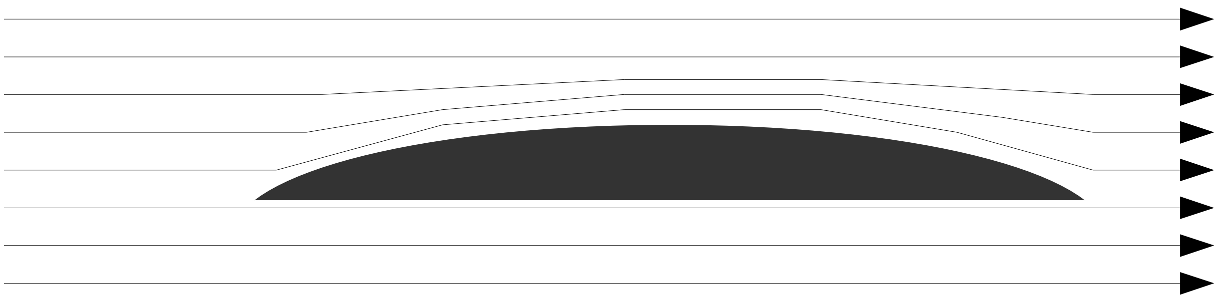
$$\rho g h_2 + \frac{1}{2}\rho v_2^2 + P_2 = \rho g h_1 + \frac{1}{2}\rho v_1^2 + P_1$$

$$\rightarrow \frac{1}{2}\rho v_2^2 + P_2 = P_1 \rightarrow v_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

Image: Pitot tube on helicopter by Zátonyi Sándor
https://en.wikipedia.org/wiki/File:Pitot-cs%C5%91_helikopteren_4.jpg#/media/File:Pitot-cs%C5%91_helikopteren_2.jpg

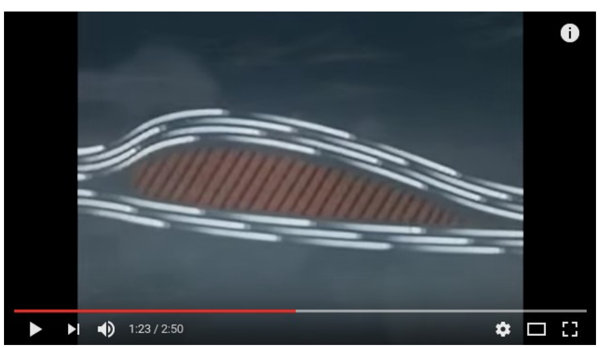
Some Applications

Lift on airplanes



The curvature of the flow causes the pressure above the wing to be lower than below the wing

→ the wing is lifted upwards.



Watch:
https://www.youtube.com/watch?v=uUMlnIwo2Qo

Some Applications

And many more….

Additional Resources

- Fluid Statics in “College Physics” Chapter 12.1-12.2
  http://cnx.org/contents/Ax2o07Ul@9.39:Y3wj6FHT@3/Introduction-to-Fluid-Dynamics
  - Suggested Problems: *Questions (Chap 12): 1, 4, 9, 11, 12, 14, 17*
    Problems (Chap 12): 1, 3, 5, 11, 13, 19, 23