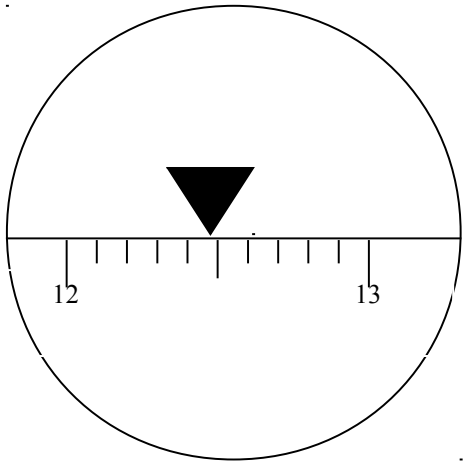
Math Review

- Measurements
- Significant Figures
- Scientific Notation
- Units and Conversion
- Graphs
- Solving Equations

Stefan Bracher

… why 2 cm and 2.0 cm is not the same



- It is not possible to make an exact measurement.
- The last written digit is the estimated digit

→ 2 cm : Anything between 1 and 3 cm
→ 2.0 cm: Anything between 1.9 and 2.1 cm

Significant Figures

… all known digits + the estimated digits

A number is significant when it is:

- not a zero — *123 → 3 significant figures*
- a zero between non-zero digits — *1001 → 4 significant figures*
- a zero after a-non zero on the right of the decimal point or on the left of the decimal point — *1.00 → 3 significant figures*; *100. → 3 significant figures*
- in the coefficient of a scientific number — *1.00×10^3 → 3 significant figures*

A zero is not significant when it is:

- on the left of all non-zero digits — *0.03 → 1 significant figure*
- on the right of all non-zeros in a number without decimal point — *100 → 1 significant figure*

Significant Figures

Exact Numbers

Counted:
12 eggs, 3 donuts

Definitions:
1 foot = 12 inches
1 meter = 100 cm

Calculation rules

Always ignore exact numbers

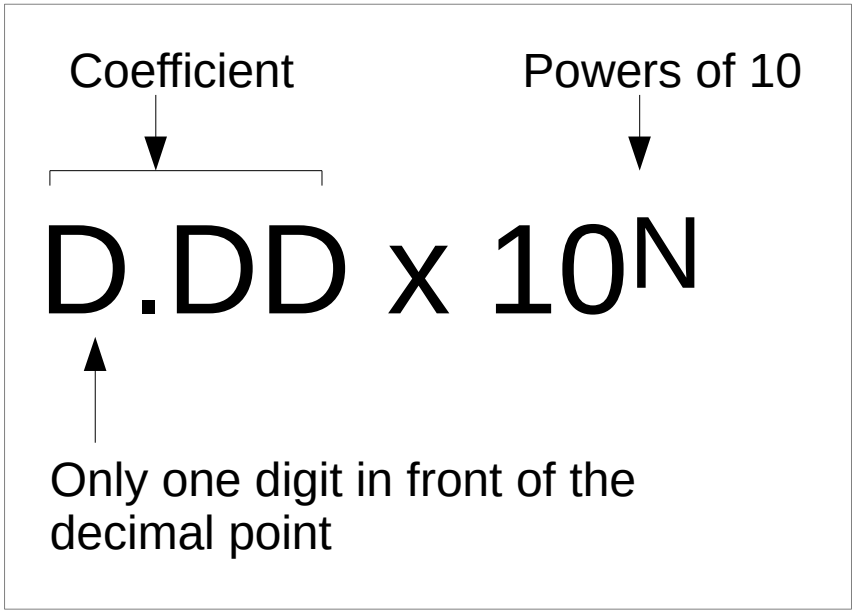
Addition / Subtraction
Give the result with the fewest decimals

1.00 + 2.3 = 3.3
5.5 - 0.50 = 5.0

Multiplication / Division
Give the result with the fewest significant figures.

2.00 x 2.0 = 4.0
4 / 2.00 = 2

Scientific Notation



Additional Rules

$10^A x 10^B = 10^{(A+B)}$
$10^A / 10^B = 10^{(A-B)}$

Scientific notation → Ordinary number

N > 0 → Shift decimal point right N-times
N < 0 → Shift decimal point left N-times

3.31×10^4 → 33100
2.1×10^{-3} → 0.0021

Ordinary number → Scientific notation

Shift decimal point left N-Times → N > 0
Shift decimal point right N-Times → N < 0

33100 → 3.31×10^4
0.0021 → 2.1×10^{-3}

Calculator

Use “EE” or “Exp” buttons, avoid “10^”

Units – The SI-System

- Length: Meter [m]
- Mass: Kilogram [kg]
- Time: Second [s]
- Temperature: Kelvin [K]
- Current: Ampere [A]

Examples:

Prefixes:

tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Units – The SI System vs. the US System

SI: International System of Units

US: United States customary units
(similar in most cases to the old Imperial System)

	SI	US	Conversion
Length:	Meter [m]	foot [ft]	1.000 ft= 0.3048 m
Force:	Newton [N]	Kilo-pound [kip],[kips]	1.00 kip = 4450 N

Unit Conversion (Unit analysis)

$$\text{New Unit} = \text{Old Unit} \times \frac{\text{New Unit value}}{\text{Old Unit value}}$$

May be repeated several times

Keep same amount of SF

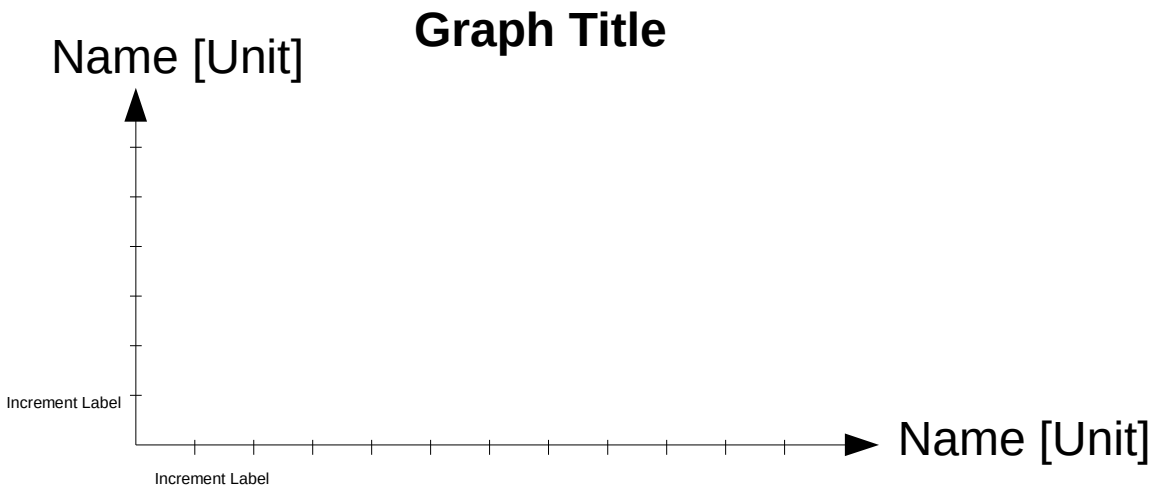
Example 1: Convert 300 m to km

$$300\ \text{m} \times \frac{1\ \text{km}}{1000\ \text{m}} = 300 \times 1 / 1000\ \text{km} = 0.3\ \text{km}$$

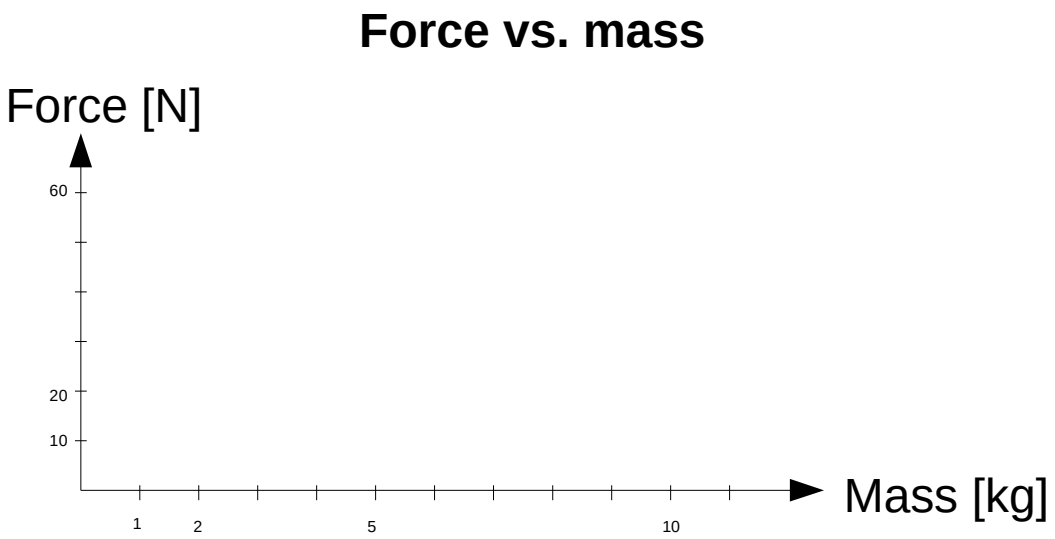
Example 2: Convert 5.0 km/h to m/s

$$5.0\ \frac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \ldots$$

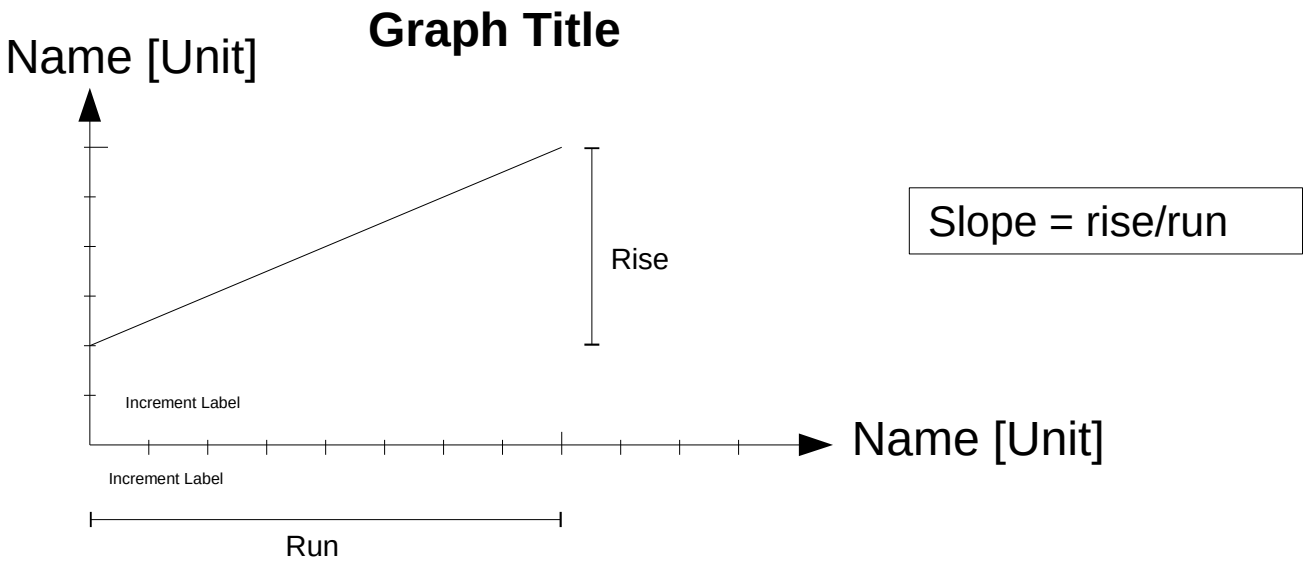
Graphs



Example

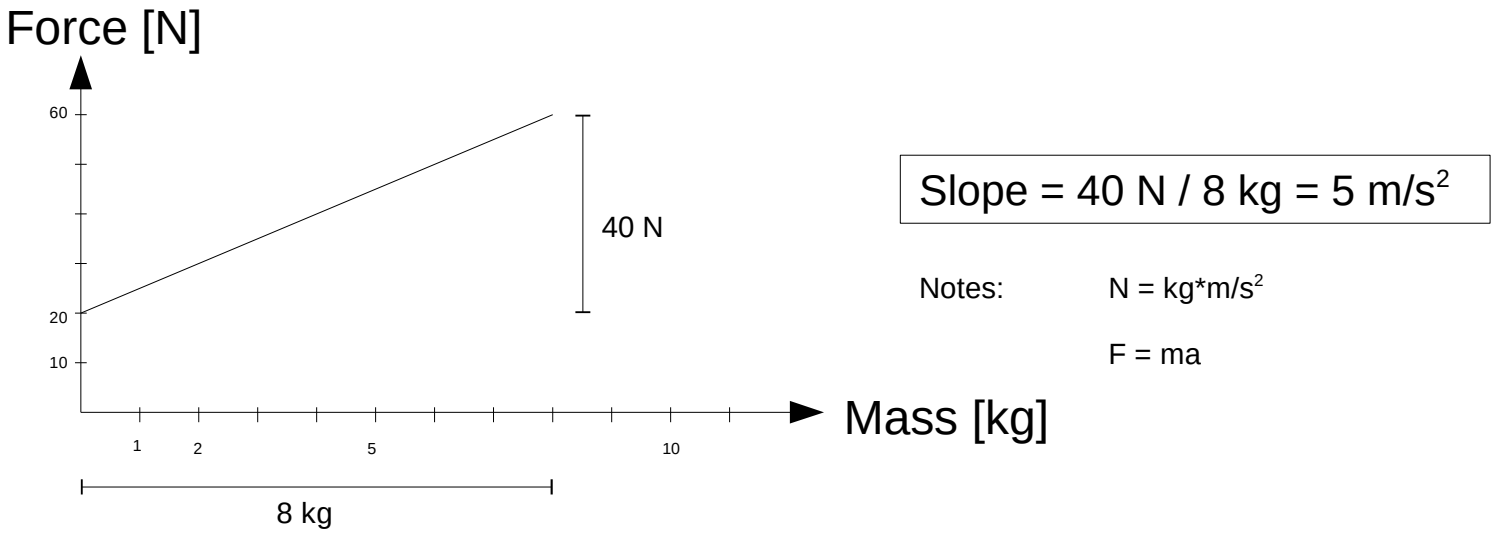


Graphs - Slope



Slope = rise/run

Example

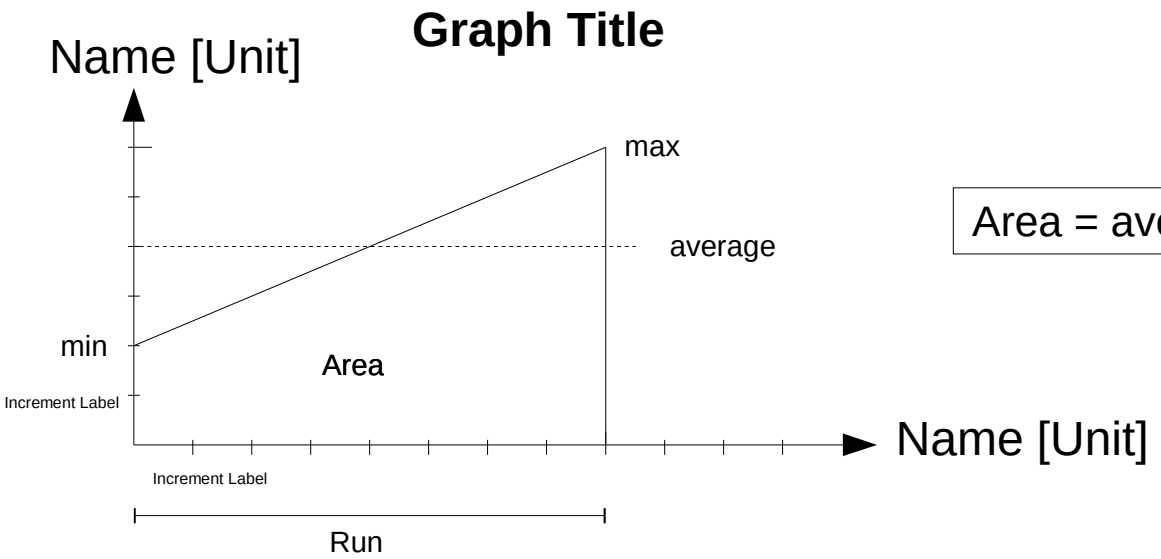


Slope = 40 N / 8 kg = 5 m/s^2

Notes: $N = kg*m/s^2$

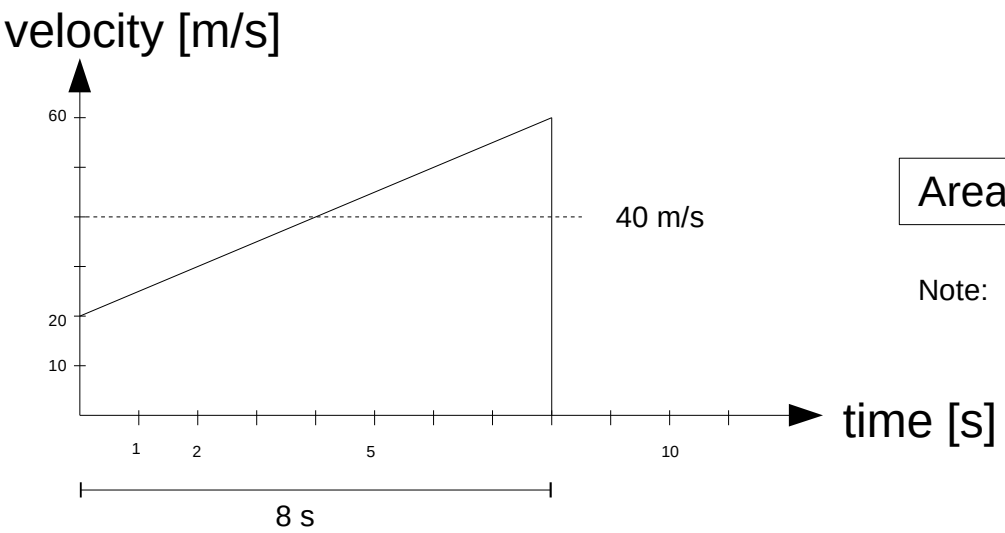
$F = ma$

Graphs – Area under the curve



Area = average * run

Example



Area = 40 m/s * 8s = 320 m

Note: s = v*t

Solving equations – 1 unknown

Idea: Isolate the unknown by executing the same operations on both sides of the equation

Example:

Equation	Operation
$12x + 3 = 27$	
$12x + 3 - 3 = 27 - 3$	- 3
$12x = 24$	
$12x/12 = 24/12$	/ 12
$x = 2$	

Solving equations – Quadratic Equation

Idea: 1. Bring the equation into the form $ax^2 + bx + c = 0$

2. Solve using a calculator or $x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example:

Equation	Operation
$12x + 3.0 = 27x^2$	$- 27x^2$
$- 27x^2 + 12x + 3.0 = 0$	a=-27, b=12, c=3.0
	EL-520X: Mode->EQN->Quad

$X_1 = - 0.18$

$X_2 = 0.62$

Solving system of linear equations – several unknowns

Idea:

1. For each unknown, find one independent equation
2. Bring the system of equations in standard form

$$A_1x + B_1y + C_1z = D_1$$
$$A_2x + B_2y + C_2z = D_2$$
$$A_3x + B_3y + C_3z = D_3$$

3. Solve using the calculator or a math method (Substitution, Gauss, Matrix inversion, ...)

Example:

1. System of equations:

$$3x = -2y$$
$$y + z = 5$$
$$x + -3 + z = 0$$

2. Standard Form:

$$3x + 2y + 0z = 0$$
$$0x + 1y + 1z = 5$$
$$1x + 0y + 1z = 3$$

3. EL-520X: Mode->EQN->3-VLE

a1=3, b1=2, c1=0, d1=0
a2=0, b2=1, c2=1, d2=5
a3=1, b3=0, c3=1, d3=3

X = -0.8
Y = 1.2
Z = 3.8

Additional Resources

Further reading

- Physical Quantities and Units in “College Physics”
  http://cnx.org/contents/Ax2o07Ul@9.4:EC6WBNqn@7/Physical-Quantities-and-Units
- Measurements and Density in „College Chemistry“
  http://cnx.org/contents/havxkyvS@9.110:GCPSnOuw@5/Measurements
- Exponents and Scientific Notation in „College Algebra“
  https://cnx.org/contents/mwjClAV_@5.57:Blwxgq2s@8/Exponents-and-Scientific-Notat
- Quadratic Equations in „College Algebra“
  https://cnx.org/contents/mwjClAV_@5.57:ABOAASP-@13/Quadratic-Equations
- Systems of Equations in „College Algebra”
  https://cnx.org/contents/mwjClAV_@5.57:CguG3zFm/Introduction-to-Systems-of-Equ

Problems

- “Scientific Math Bingo”
  http://stefan.bracher.info/files/physics/scientific_bingo_math.htm