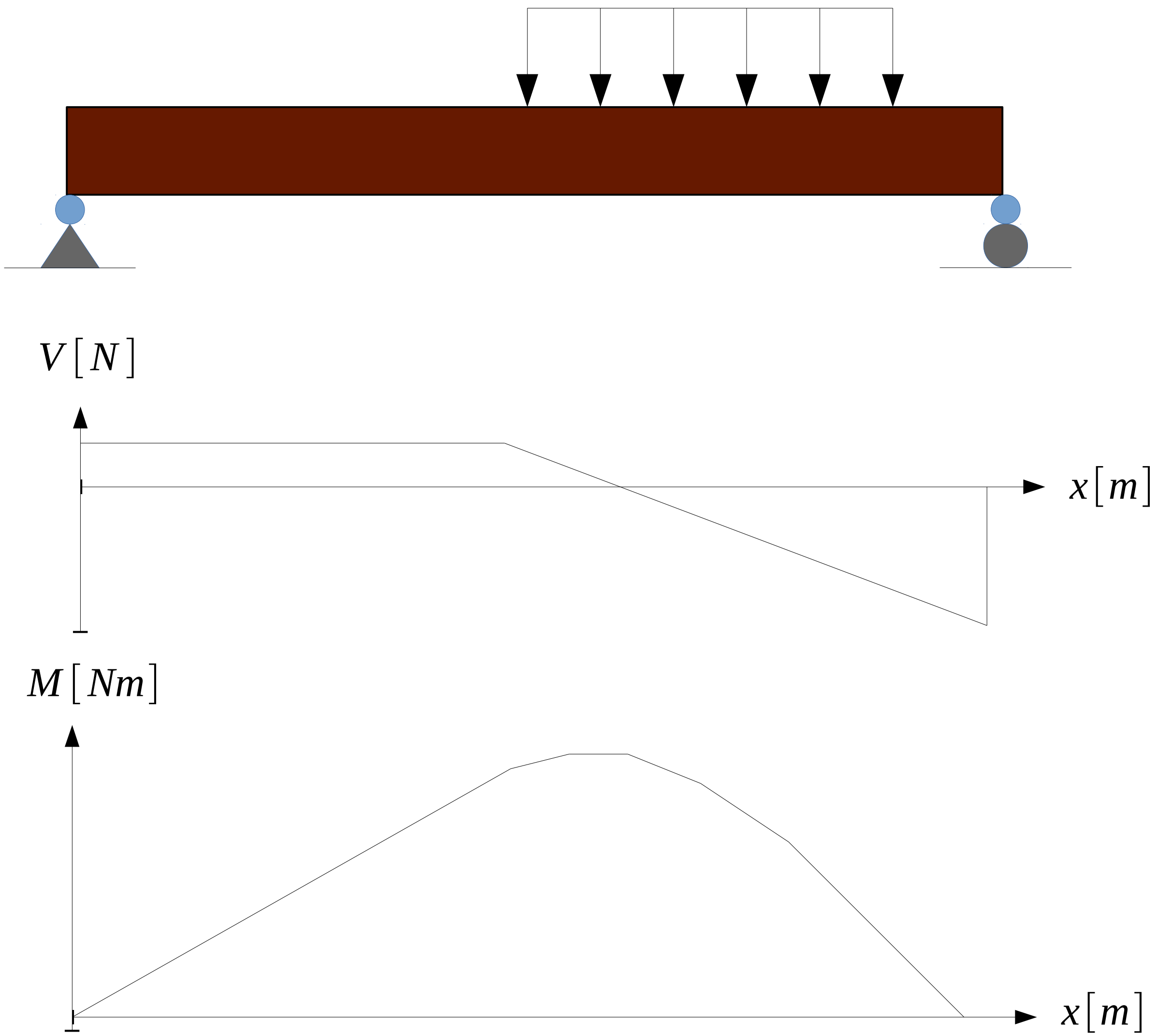
Shear Force and Bending Moment Diagrams



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Competencies

1. Draw shear force and bending moment diagrams for point loads and distributed loads

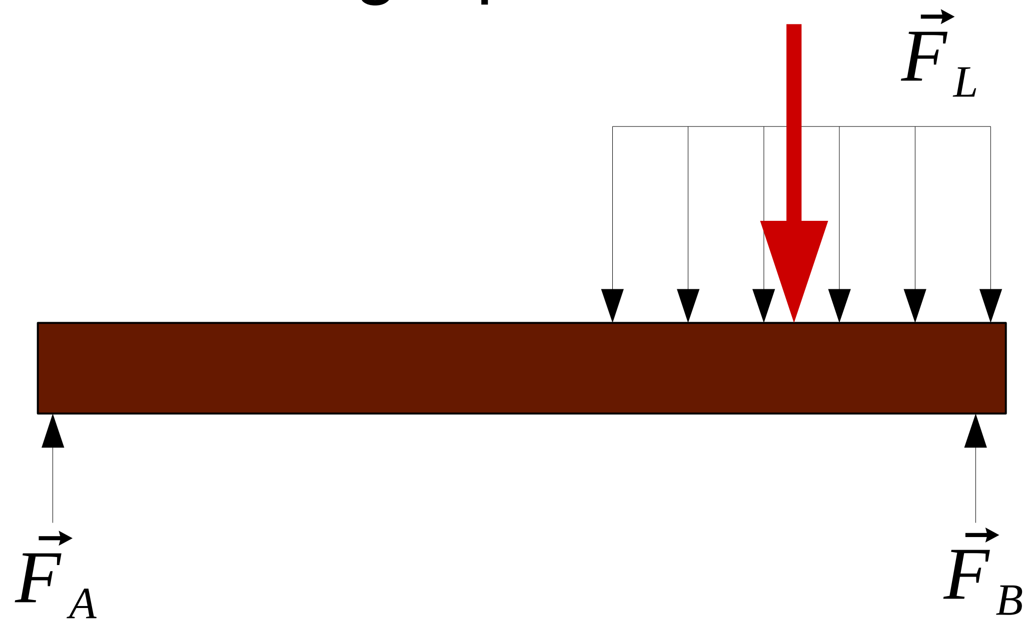
2. Recognize the position of maximum moment as the location of zero shear force.

3. Recognize the moment diagram as the area under the shear force diagram.

Beam: A horizontal structural element with vertical loads

Concentrated load: An external load that is acting on a single point.

Distributed load: A external load that is distributed over some area instead of acting on a single point.



External loads cause internal forces.

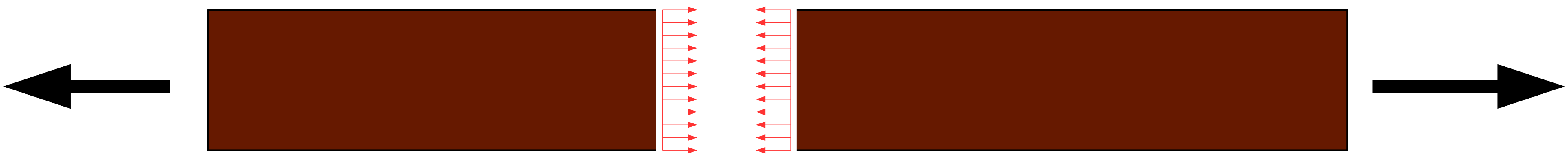
Internal forces cause structural elements to fail.

→ If we can calculate the maximal internal forces, we can look up in a table what structural element is needed (material, cross section area, shape) to withstand those forces.

Types of internal forces

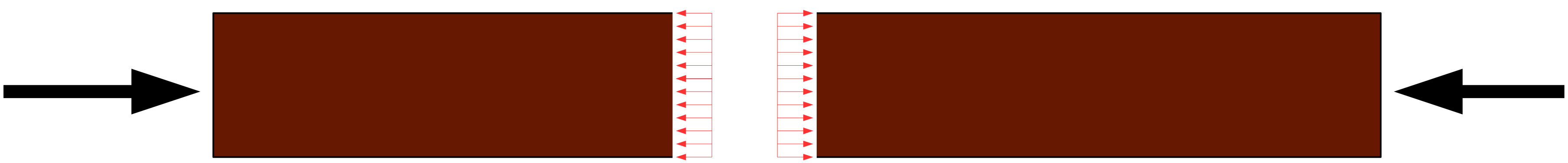
Tension: Material is pulled apart

Cause: Axial loads



Compression: Material is squeezed

Cause: Axial loads

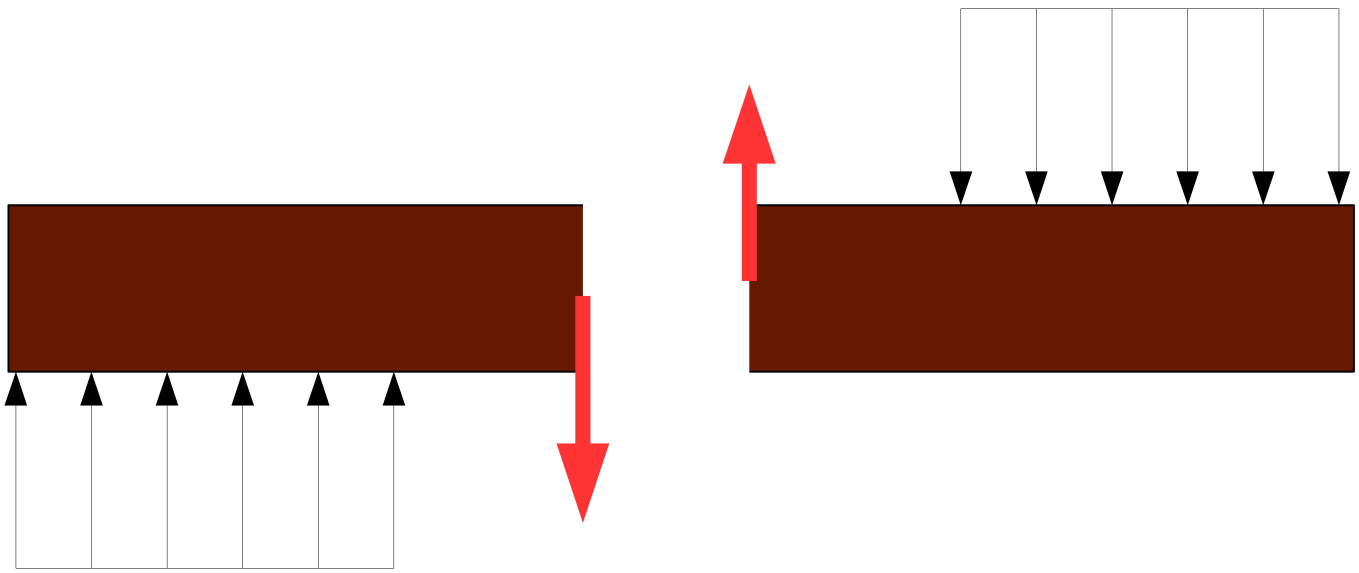


Hint: Think what force would be needed to keep each chunk of the structural element in static equilibrium.

Types of internal forces

Shear: Material is torn apart vertically

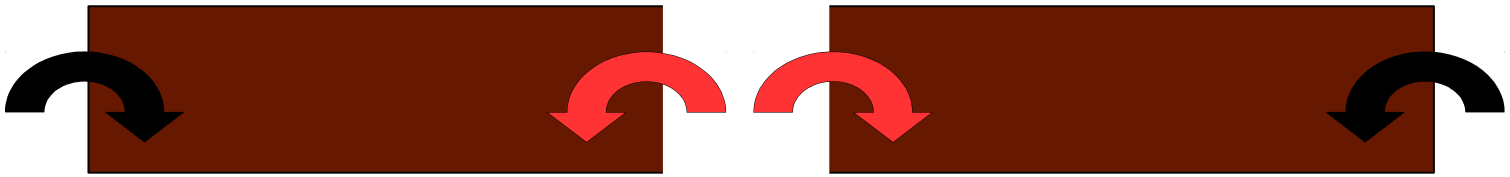
Cause: Vertical loads



Again: Think what force would be needed to keep each chunk of the structural element in static equilibrium.

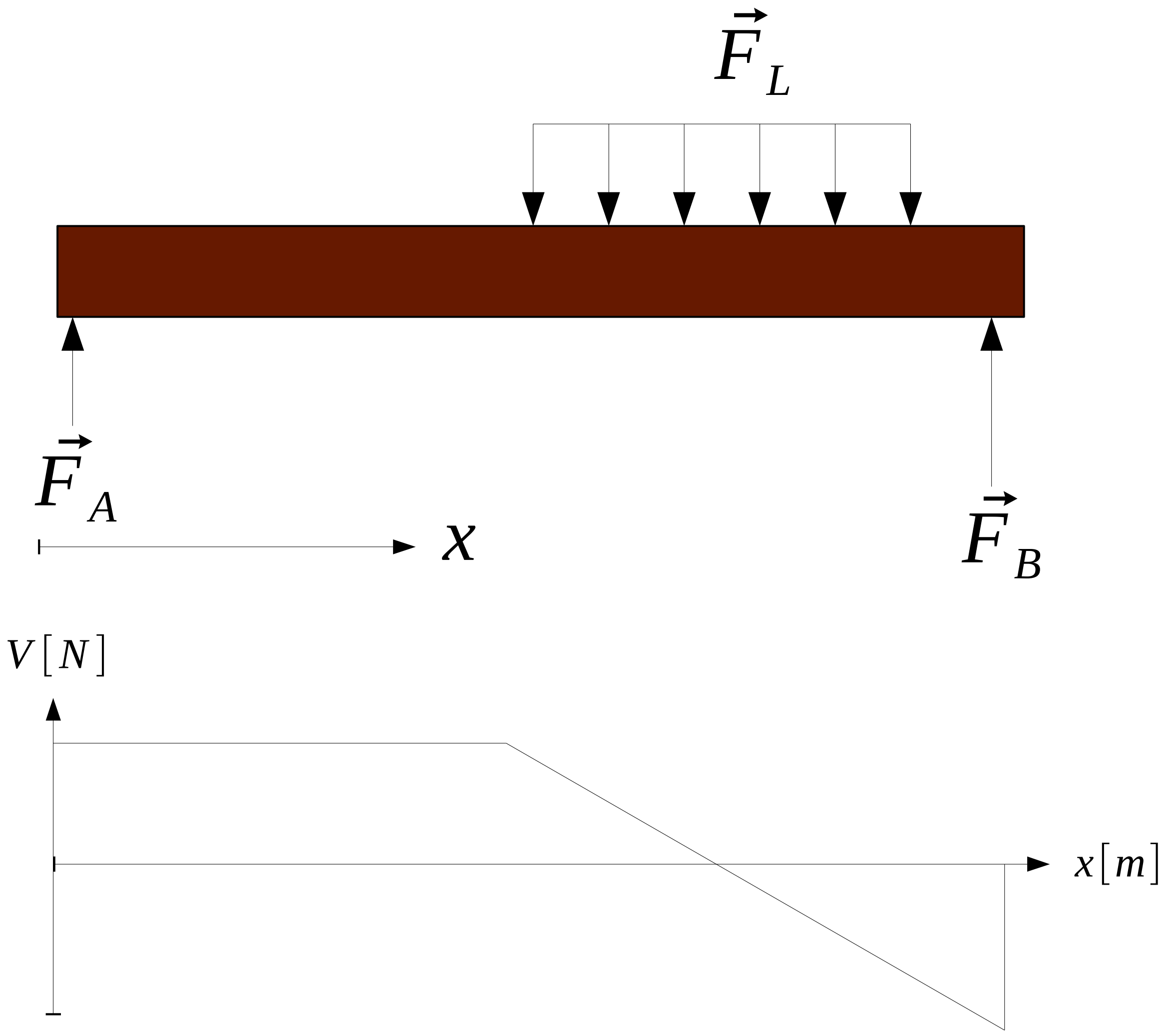
Bending Moment: Material is bent

Cause: Torques (caused by types of forces)



Again: Think what force would be needed to keep each chunk of the structural element in static equilibrium.

Shear Diagram: Plotting the shear force V as a function of the distance x from the left side of the beam



How to draw the shear force diagram:

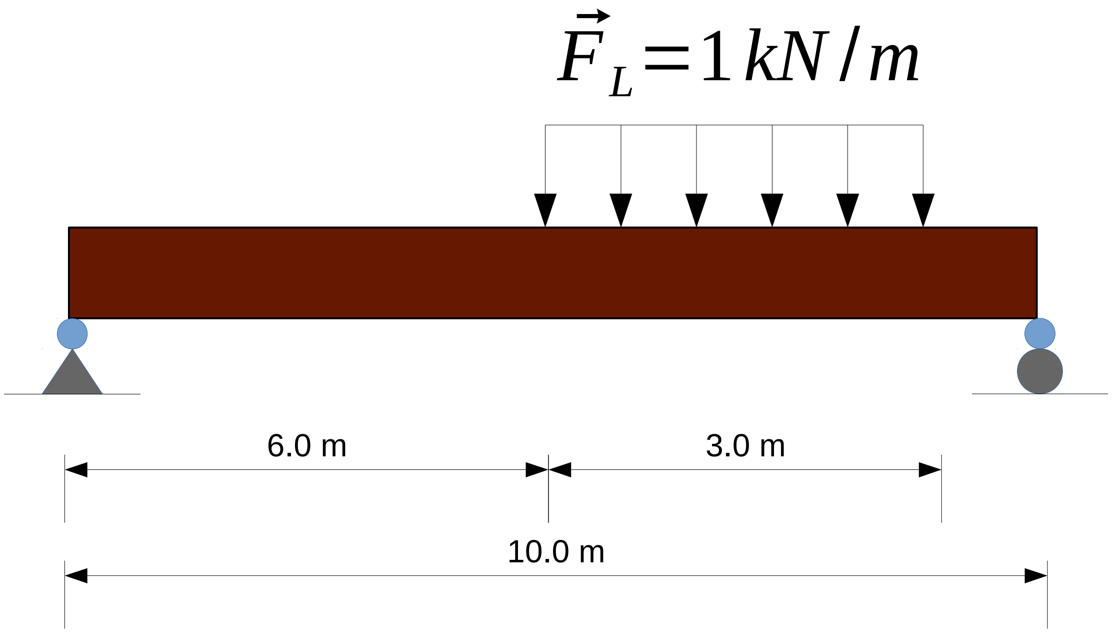
1. Calculate the reaction forces

1.1 Replace distributed loads by equivalent point loads
1.2 Draw the free body diagram
1.3 Use $\vec{\tau}_{net}=0$ *and* $\vec{F}_{net}=0$

2. Start at the left with V=0 then move along the beam to the right:
   - Whenever you encounter an upward vertical load, increase V by the amount of the vertical load.
   - Whenever you encounter an downward vertical load, decrease V by the amount of the vertical load.

Shear Force diagram

Example: A beam (negligible mass) is loaded as shown.
Draw the Shear Force Diagram (SFD)



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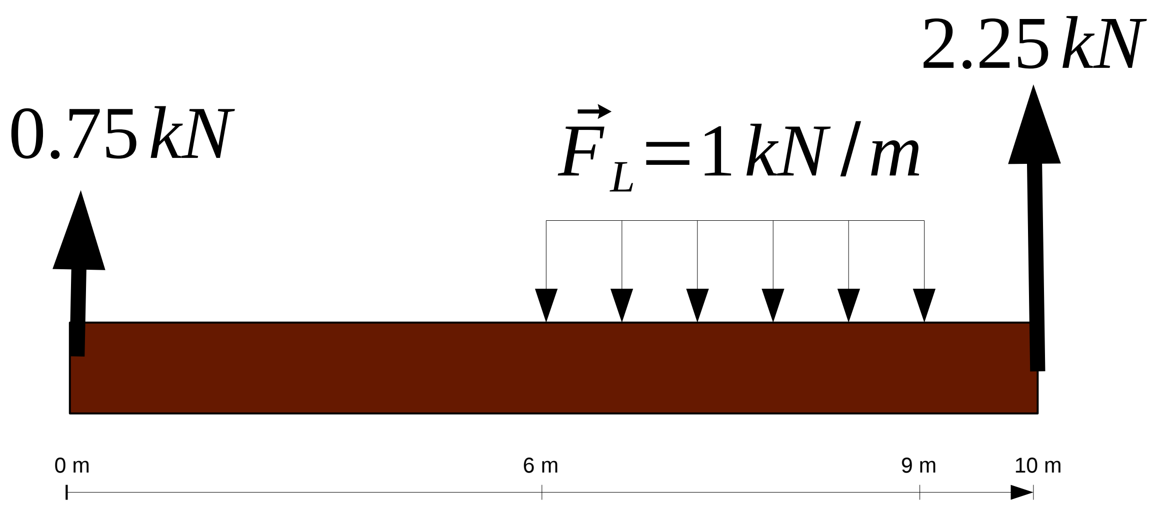
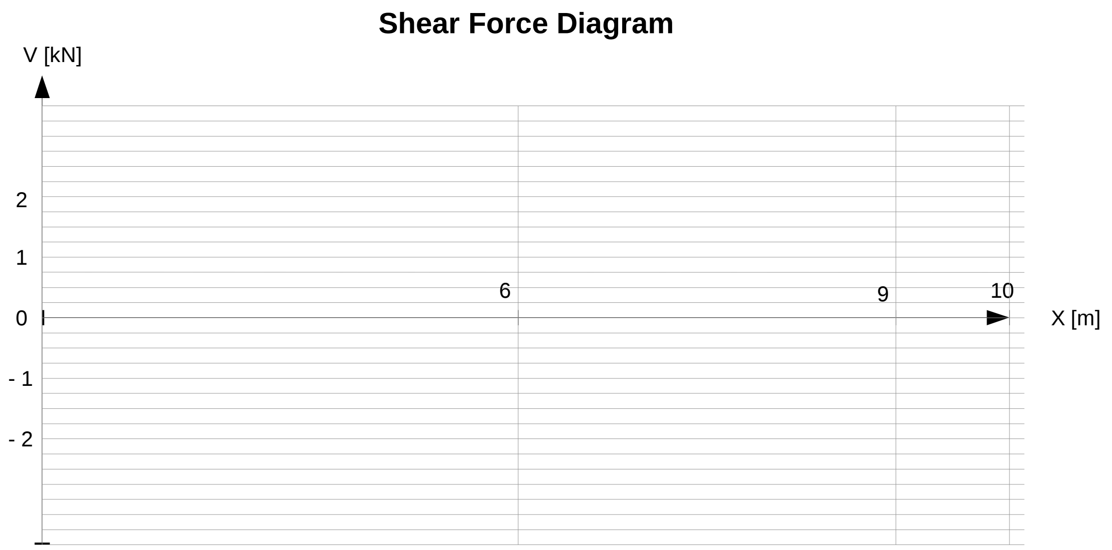
Shear Force Diagram

… Example continued





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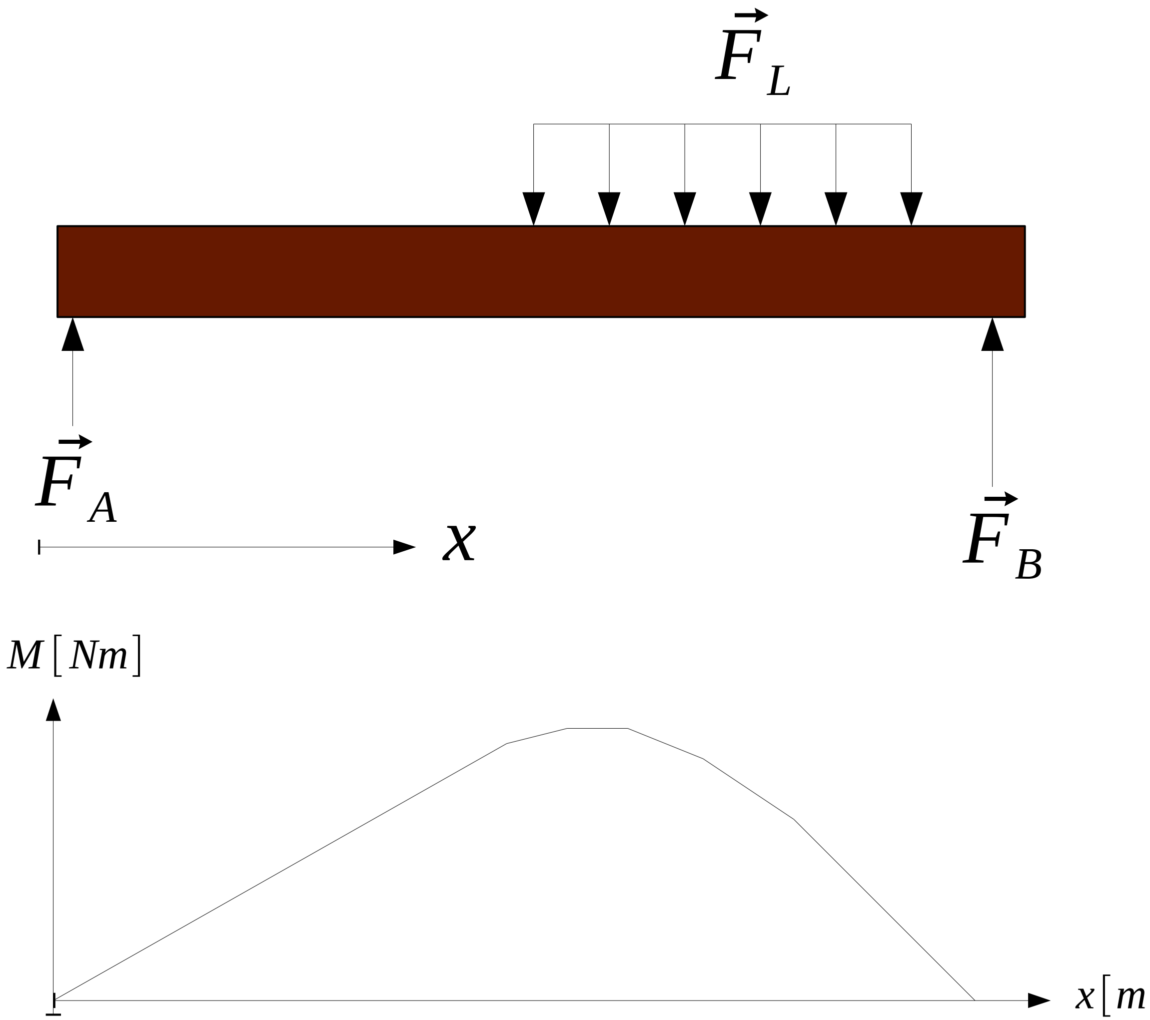
Bending Moment Diagram:
Plotting the bending moment M as a function of the distance x from the left side of the beam



Bending Moment Diagram

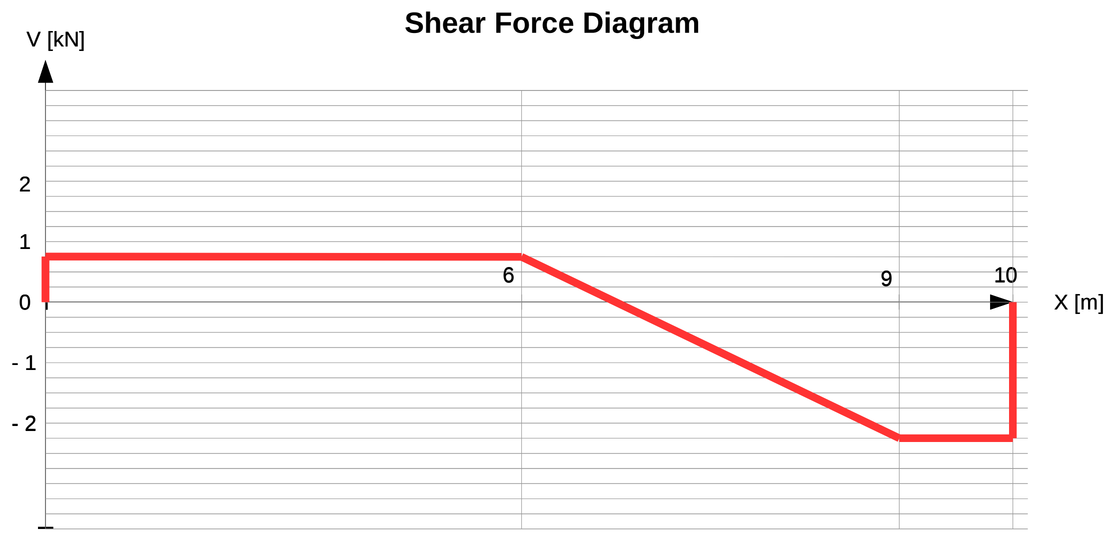
How to draw the bending moment diagram:

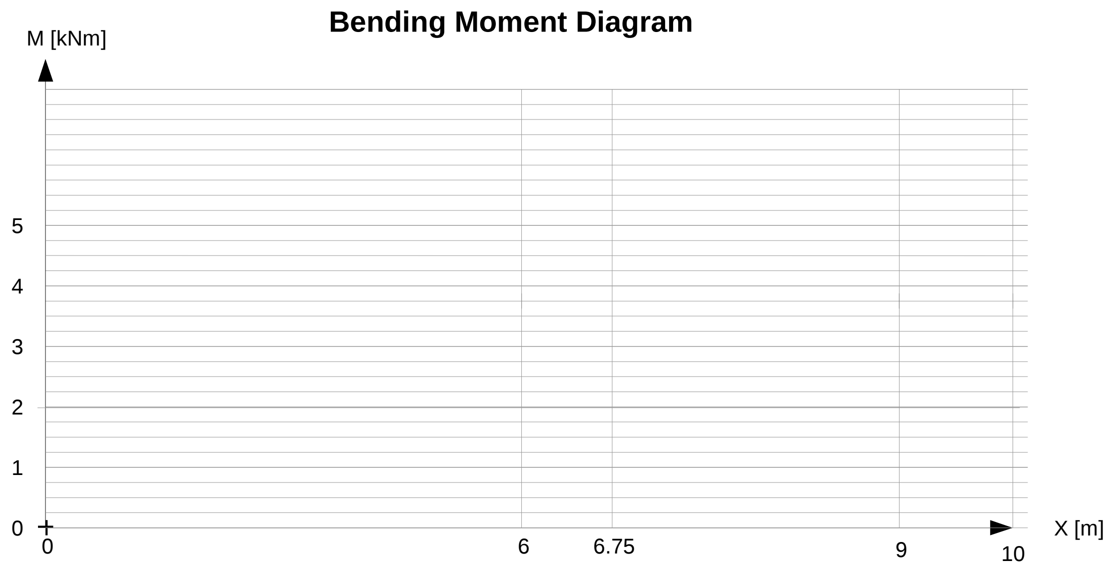
1. *Draw the shear force diagram (SFD)*

2. Start at the left with M=0 then move along the beam to the right. Calculate the total area under the shear force diagram curve from the left:
   - If the area is above the x-line, count it as positive.
   - If the area is below the x-line, count it as negative

Bending Moment Diagram

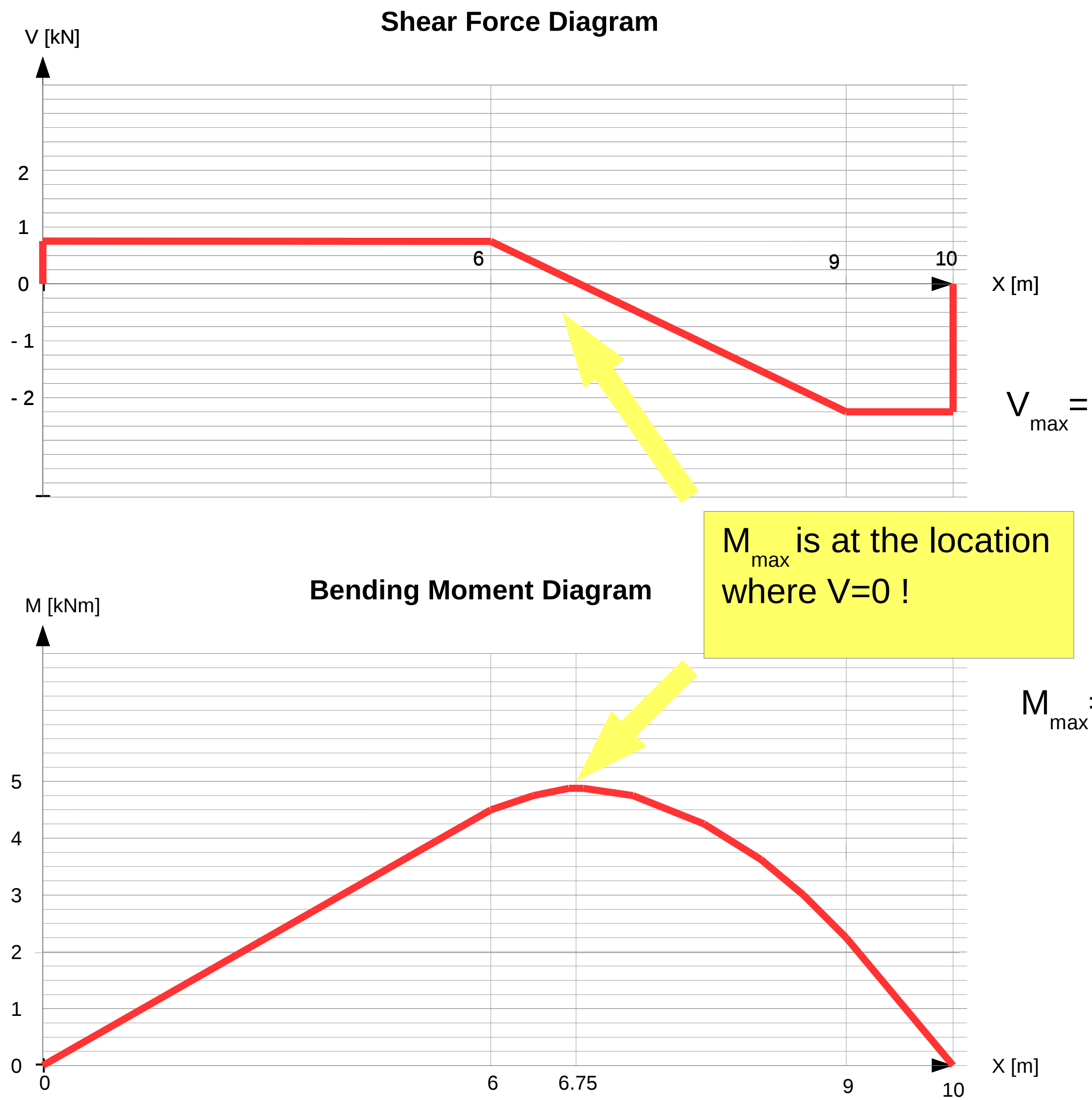
Our Example:





Bending Moment Diagram

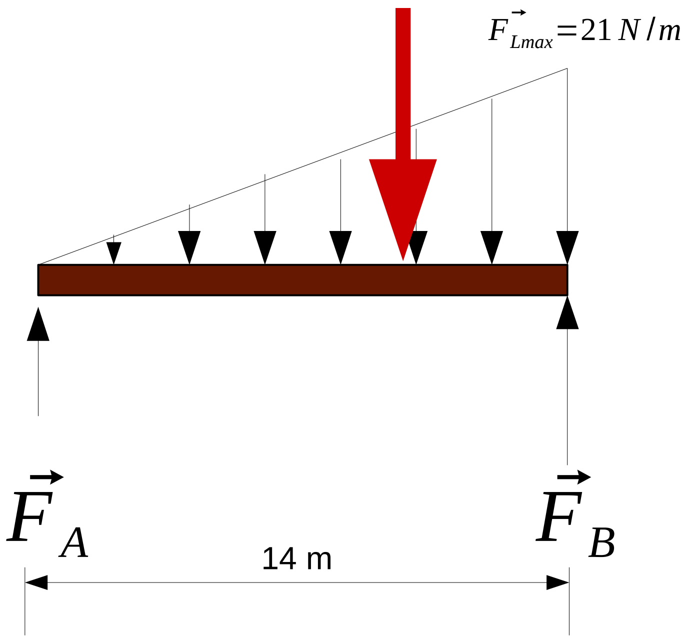
Our Example:



V_{max} = -2.25 kNm

M_{max} = 4.78 kNm

How to solve a problem with a triangular load:



1. Don't panic

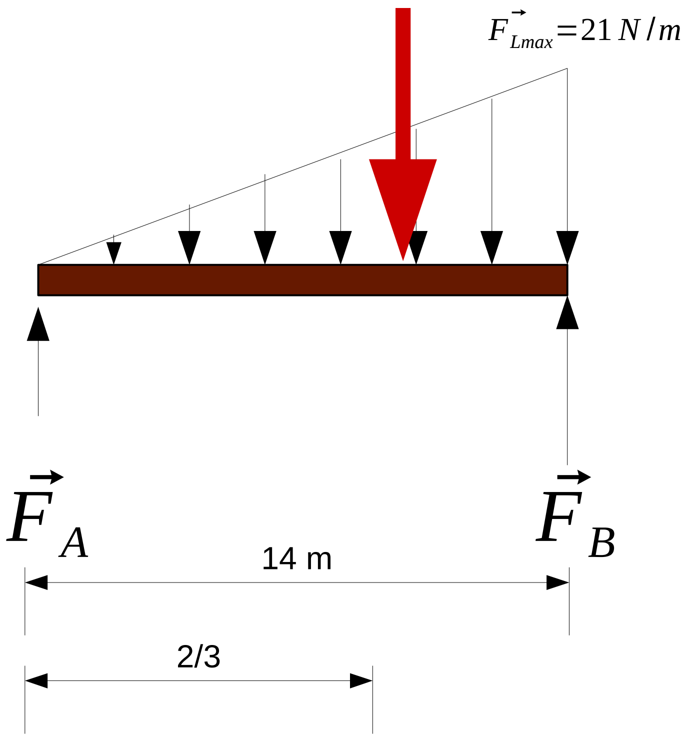
2. Calculate the equivalent load

3. Calculate the reaction forces

4. Calculate **some** points on the Shear Force Diagram

5. Calculate **some** points on the Bending Moment Diagram using the equivalent force **LEFT OF THE POINT.**

How to solve a problem with a triangular load:

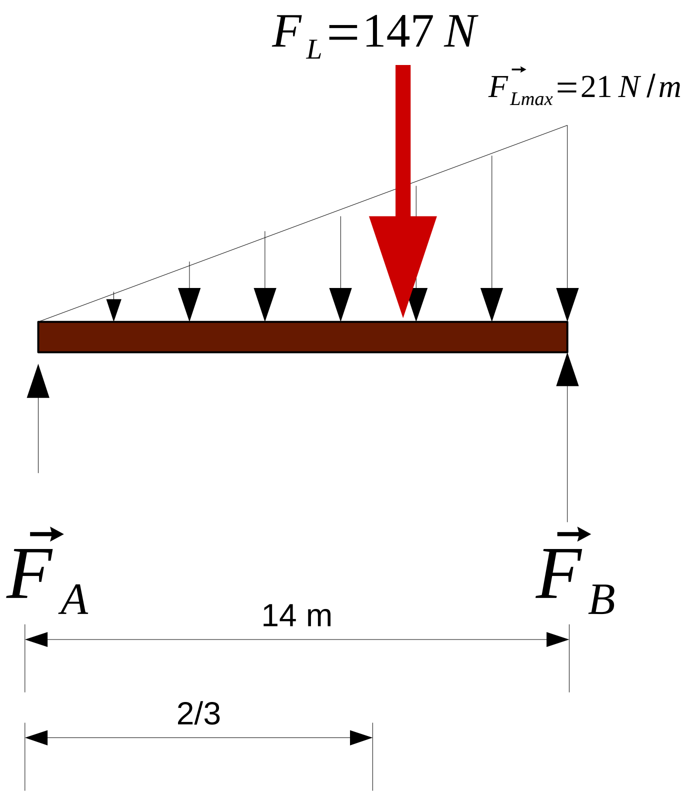


1. Don't panic

2. Calculate the equivalent load

$$F_L = 147\ N$$

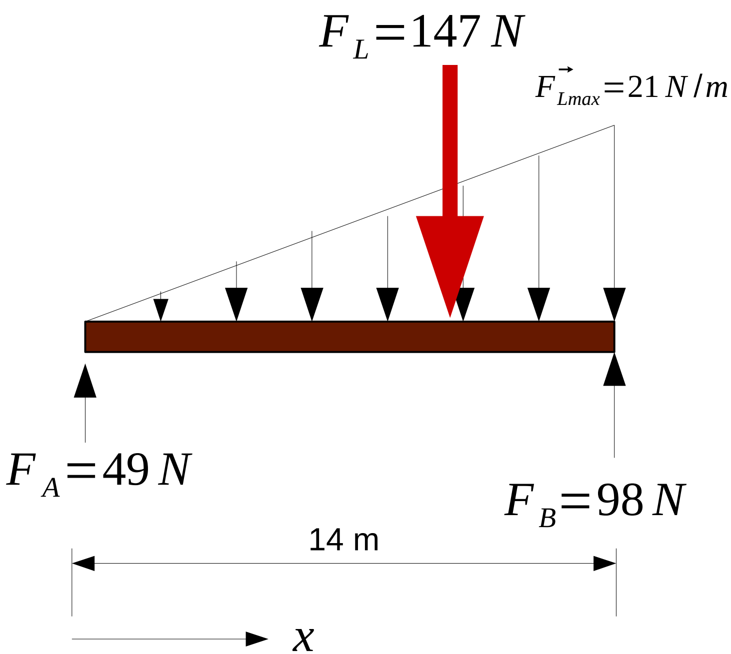
How to solve a problem with a triangular load:



3. Calculate the reaction forces

$$F_A = 49\,N \qquad F_B = 98\,N$$

How to solve a problem with a triangular load:



4. Calculate **some** points on the Shear Force Diagram

Left end of the diagram:
V(x=0m) : Jump from 0 N to + 49 N

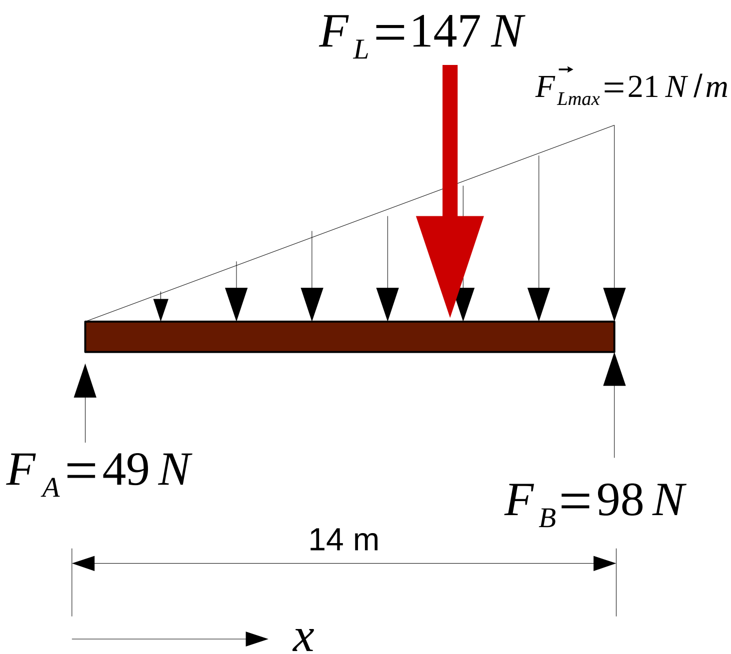
Right end of the diagram:
V(x=14m) : Jump from -98 N to 0 N

Random point x = 7
Load at x:
L(x)=21 N/m / 14 m * x = 10.5 N

Total load left of x:
LL(x)=1/2*7m*10.5 N = 36.75 N

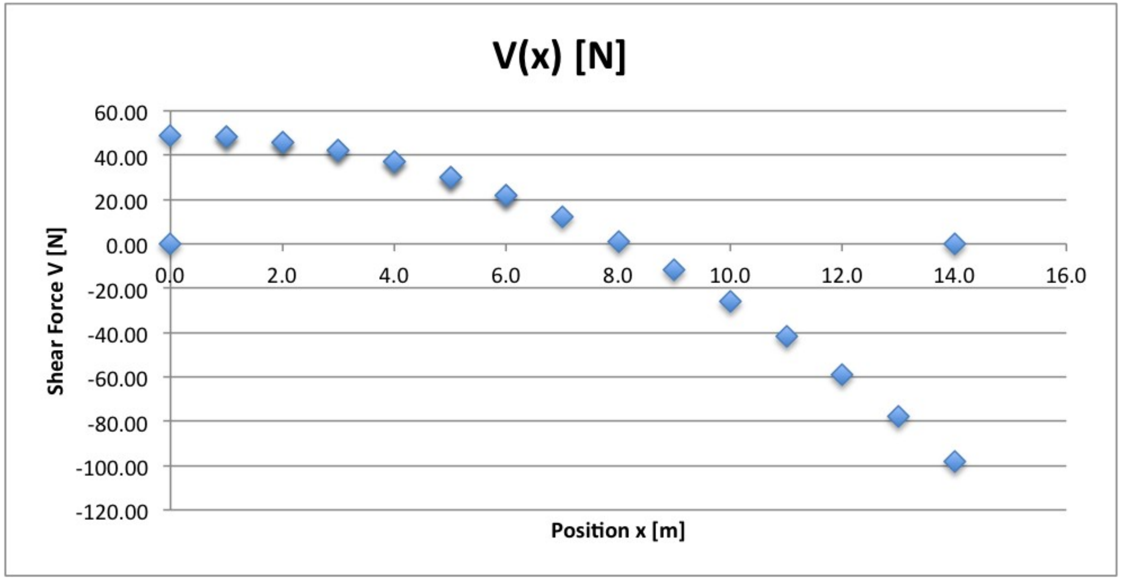
Shear at location x (for x < 14):
V(x) = V(0)-LL(x) = 49 N – 36.75 N
= 12.25 N

How to solve a problem with a triangular load:



4. Calculate **some** points on the Shear Force Diagram

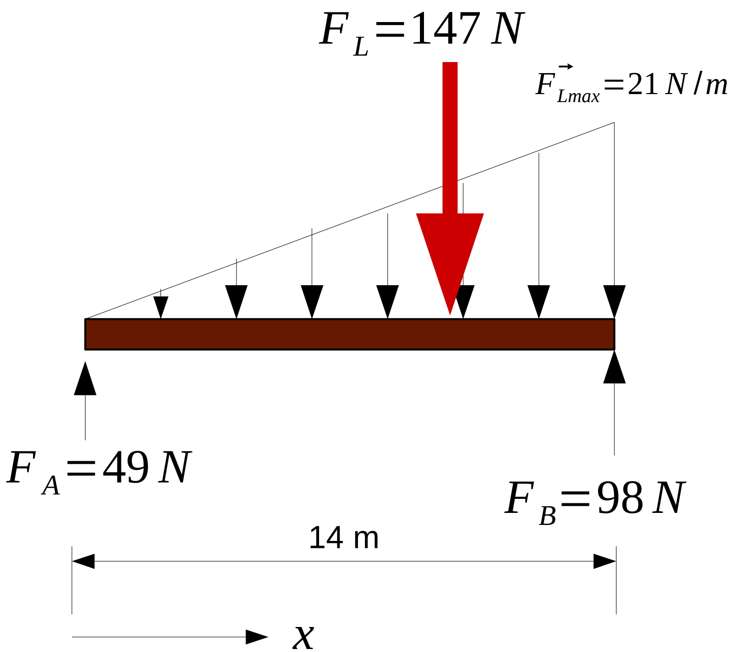
Result Excel:

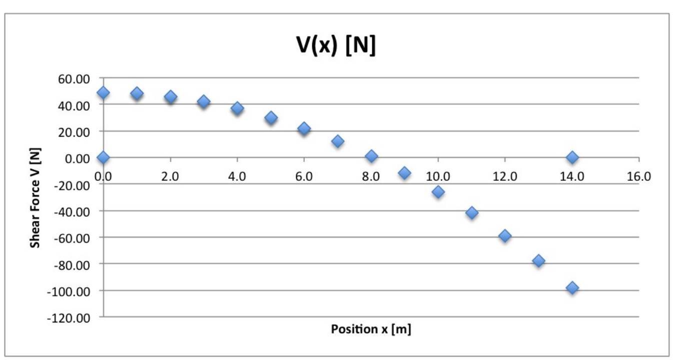


In case you need every single point →

Analytical Result:
$V(x)=49-0.5*21/14*x*x=49-0.75\,x^2$

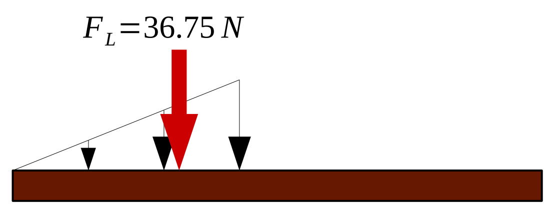
How to solve a problem with a triangular load:



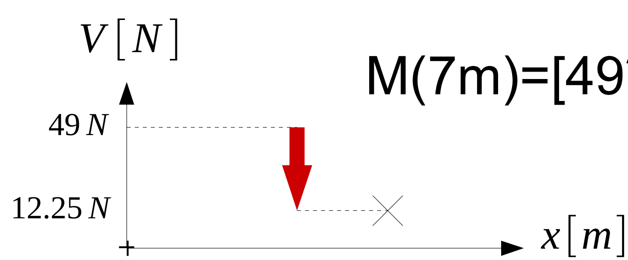


5. Calculate <u>**some**</u> points on the Bending Moment Diagram using the equivalent force **LEFT OF THE POINT.**

Random point x = 7
Total load left of x:
LL(x)=1/2*7m*10.5 N = 36.75 N



M(x)=Area under V(x):
Calculate the area under V(x) at x=7 using an approximated shear force diagram:



M(7m)=[49*2/3+12.25*1/3]N*7m
= 257.25 Nm

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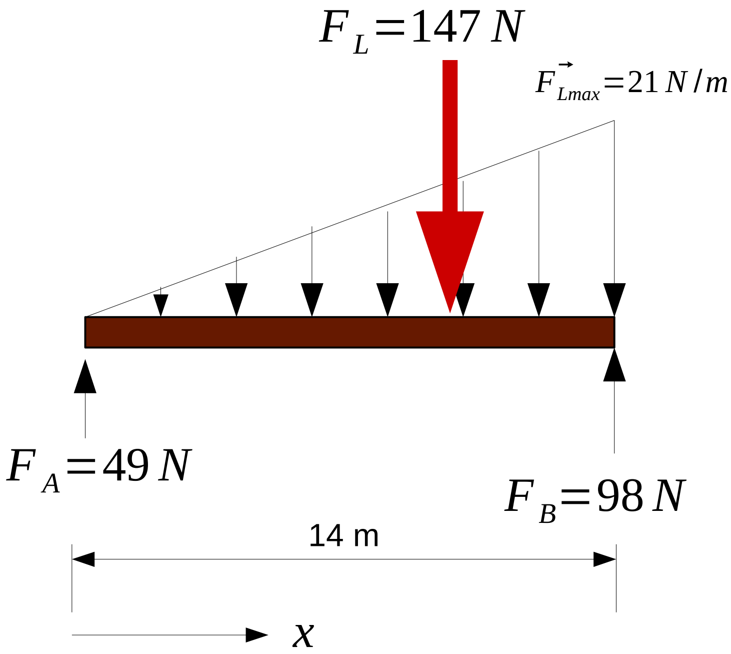
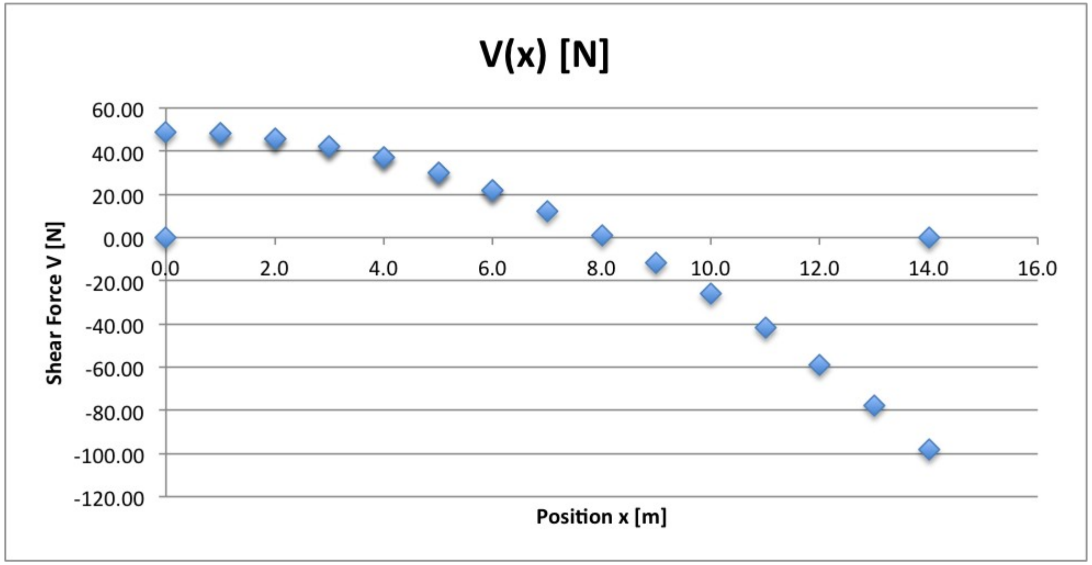
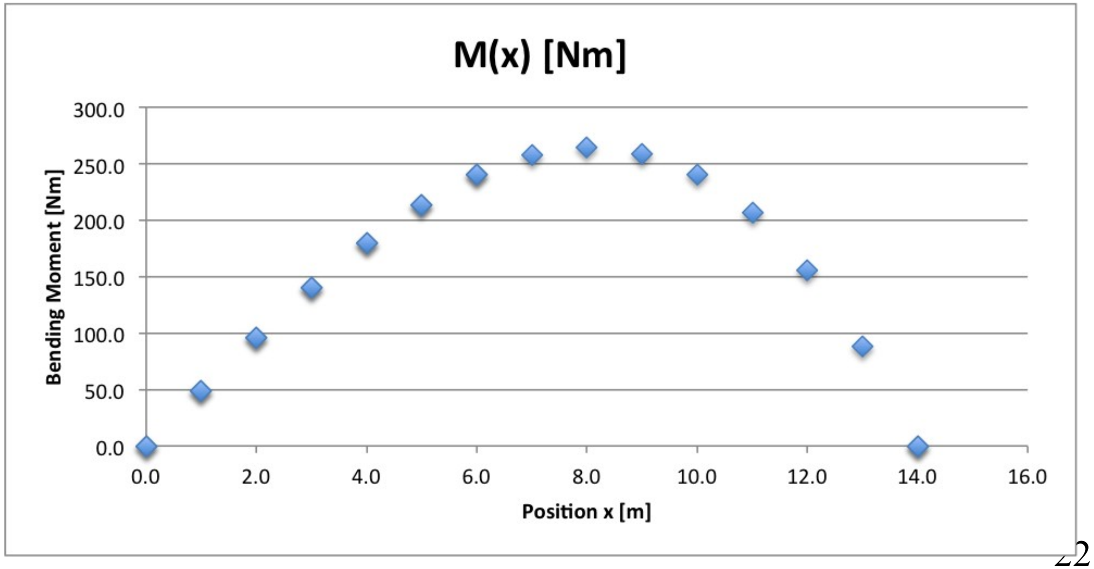
How to solve a problem with a triangular load:







If you knew calculus …

Once we found V(x) Here (Page 20): $V(x)=49-0.75\,x^2$

M(x) could be calculated as the integral of V(x) (which is equal to calculating the area under V(x) as:

$$M(x)=\int_0^x 49-0.75\,x^2\,dx=49\,x-0.75\frac{x^3}{3}$$

Free online beam calculator
https://skyciv.com/free-beam-calculator/

→ Create your own practice problems with solutions.
(Or “cheat” when doing homework)